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U.S. livestock production and factor demand: a multiproduct dynamic dual approach

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CHAPTER 1. INTRODUCTION

Problem Setting

Almost all governments in the world are involved in regulating agriculture, though for various reasons. For example, an important goal of the U.S. farm policy is to maintain a reasonable level of income to farm population through a battery of supply management programs. Despite the recent efforts to 'free' the world agriculture from government intervention through the General Agreement on Tariffs and Trade (GATT) negotiations, U.S. agriculture continues to enjoy various forms of government protection. The estimated net government expenditures on such measures peaked at $26.94 billion in 1986 as compared to a meager $3.83 billion in 1980. Though many farm programs are commodity-specific, important interdependencies do exist between commodity markets, and some policies have simultaneous effects on several markets. To integrate these various externalities into a strict quantification of the economic impacts of agricultural intervention is rather difficult. Thus, the economic model builders tend to ignore them, appealing to the insignificance of such cross effects in interrelated sectors.

Several models have been constructed and estimated to simulate and explore the consequences of various farm programs. The simplest assessments of agricultural policy are those of single (commodity) sector studies. For example, Harling and Thompson (1985) for poultry and eggs in Canada, the United Kingdom and West Germany, Anderson (1985) for cheese in the U.S., Hammond and Brooks (1985) for dairy in the U.S., and Otsuka and Hayami (1985) for rice in Japan. Multi-sectoral studies like Bale and Lutz (1981) typically apply the simple partial equilibrium approach to several sectors simultaneously, but without explicitly modeling the interactions between them. Its main payoff in principle is the comparability between sectors that it lends rather than its ability to assess the agricultural policy overall. Gardner (1985), and Tyers and Anderson (1986) provide further examples of multi-sectoral study of agricultural policies in a partial equilibrium framework.

Partial equilibrium analysis of impacts of farm policies is likely to be misleading because of the large leakages out of and into agriculture. Rosine and Helmberger (1974) took one of the early steps towards general equilibrium modeling of the macroeconomic and inter-sectoral consequences of U.S. agricultural policy. Simulating the abolition of the U.S. farm policies in 1970, they find that consumers and taxpayers would lose $4.8 billion and producers would gain $2.7 billion. Recently, computable general equilibrium (CGE) modeling has become an attractive means of assessing the impact of policy interventions. Harris and Cox (1984), Whalley (1985), Tyers (1985), Deardorff and Stern (1986), Adelman and Robinson (1986) and Hertel and Tsigas (1987) are but only a few of fast growing literature in this area. The number of studies following the 'true' general equilibrium approach, however, is much smaller, since most CGE models resort to calibration based on a bench-mark year data rather than econometric estimation based on time series data to arrive at the necessary parameters of the model.

The most ambitious and comprehensive general equilibrium study of agricul-

Table 1.1: Redistributional effects of U.S. agricultural programs ($ billion)

Study	Taxpayers Loss	Producer Gain	Total Social Loss
Rosine and Helmberger (1974):			
All commodities and all programs	4.8	2.7	2.2
Anderson (1985):			
Cheese and Import quotas			
1964-1979			0.07
Tyers and Anderson, (1985):			
Dairy and Sugar with all programs			
during 1980-82 (in 1980 prices)	19.2	20.4	-0.7
Gardner (1986):			
All commodities	19.2	14.2	5.0
Dairy	2.5	1.7	0.8
Beef	0.5	0.5	0.0
Feed grains	6.5	4.3	2.2

tural policy to date is the one initiated by the International Institute for Applied Systems Analysis (IIASA) in Austria through its Food and Agricultural Program (FAP) (Parikh et al., 1988). An operating version of this quantitative general equilibrium model system, called the Basic Linked System (BLS), is now being updated at the Center for for Agricultural and Rural Development (CARD) of Iowa State University, Ames. This CARD/BLS model is best suited for analyzing the policy issues in the context of international trade and resource use for food and agricultural commodities in an international setting. Results from some of the policy impact studies of U.S. agricultural policies are summarized in Table 1.1.

The importance of a CGE model with a strong theoretical underpinning as a tool for comprehensive policy analysis needs no further emphasis. A well developed production response module is an essential component of such a general equilibrium model. Supply response studies can analyze the impact of various government policies and thus, can serve to guide policy makers in their involvement in agriculture. The interdependencies between various outputs as well as the factor markets must be reckoned with for a comprehensive and meaningful policy evaluation. Schuh (1974), Shei (1978), Chambers and Just (1982), and Adelman and Robinson (1986) are among the few who recognized the importance of factor market linkage effects in policy modeling. These studies demonstrated the linkages between agriculture and the rest of the economy and leakages from and into agriculture primarily through factor markets. They emphasize that when analyzing the effects of agricultural policies on crop and livestock production, factor markets must be considered in conjunction with the output response module.

Agricultural production has some special characteristics which should influence the choice of modeling approach to be used in policy analysis. Joint production characteristics of agriculture must be recognized in supply modeling in order to assess the production effects of various price and policy changes. Studies that overlook this crucial aspect would be unable to shed any light on the relationships between inputs and specific output and the relationships between various alternative outputs (Lau and Yotopolous, 1972).

Knowledge of the ease with which a firm can change its output mix and adjust the quantities of various inputs it uses and on the relative factor-use intensities of alternative outputs is essential to assess the likely impacts of relative input and output price changes on the composition of outputs of a multiproduct firm. Policies that influence the relative price of inputs influence the composition of output as they foster the production of some commodities more than others. For instance, a policy which results in the relative price of of a given input being less than it would otherwise be, results, in general, in a larger contribution to that output which uses that input relatively intensively.

The body of literature available now on modeling efforts in livestock production is rather extensive. A critical review of the past modeling activities related to livestock supply and factor demand would illuminate the specific attributes that are important in livestock production modeling and estimation.

Hildreth and Jarret (1955) were the first ones to formalize in a mathematical framework, the intertemporal allocation of resources and production in the livestock sector. In their model, they specify the quantity of livestock and livestock product aggregates sold as dependent, among other variables, on the prices of feed grains and farm labor and on the price of livestock and livestock products. They assumed that anticipated prices are functions of current prices. Their estimates show that input price increases lead to an increase in current supply and that product price increase leads to a decline in current supply. They conclude that it is the investment demand for inventory that is the main force in explaining current supply of livestock and livestock products.

Later modeling efforts have heavily relied on conventional investment demand theory to explain the supply of livestock products. According to Grilliches (1963) investment demand theory implies that higher the expected price of the output relative to expected input price, the more will be invested. Following this approach, Reutlinger (1966) stipulated a beef supply model where investment in livestock inventory is treated as a continuous process and there exists an immediate alternative of disinvestment (slaughter) at market price. Investment and disinvestment decisions related to livestock inventory are affected either by postponing or by moving ahead the slaughter of animals. Jarvis (1969, 1974) was the first to recognize the timing of such decisions when he treated the livestock inventory as a capital asset in his model. The single animal is the focal point of interest in his approach. An optimal decision implies that that the appropriate moment of sale is that age of the animal beyond which the increase in costs exceeds the gain in income stream, all discounted to the present.

A major difficulty with the Jarvis model is that it deals with decisions concerning a single animal, without considering the constraints imposed by the herd dynamics. In reality, however, decisions are taken concerning the whole herd. This is due to the fact that the elements of the livestock herds advance through time as a system of causal chains. The set of currently available economic alternatives regarding sex and age composition of livestock herd is not independent of the preceding period herd composition. Investment takes the form of breeding and retention from slaughter. Disinvestment is in the form slaughter. It is the desired herd size which is assumed to motivate the decisions of investment and disinvestment in the different herd categories.

Nerlove et al., (1979) considered the whole herd of a producer as the focal point of interest. They assume that the producer maximizes expected present value of profits for the entire period that he/she will stay in business. Their model allows for the analytical derivation of quantitative hypotheses regarding the effects of different current and expected prices on the investment and disinvestment activities related to maintaining the desired herd size. Their results indicate that expected output price affects the investment positively and disinvestment decisions negatively. The current prices of various outputs affect the supply of various kinds of slaughtered animals differently. One important theoretical deficiency of their model is their assumption of constant per unit cost which contradicts the notion that livestock inventories are treated as capital assets.

Most econometric specifications of livestock sector still have relatively simple supply structure that use distributed lags of input and output prices, time lags, and partial adjustments to production stimuli. Seasonality, an important feature of the livestock industry, is handled with dummy variables. The lag structure in the supply block is governed by the biological restrictions imposed by the sequential phases of the livestock production process. Such known biological restrictions impose constraints on supply response and hence, should be incorporated into the behavioral equations.

Harlow (1962) developed a recursive supply structure for hog industry where a single inventory equation is specified as a partial adjustment relation which in turn governs subsequent slaughter. Modern extensions of Harlow's work include Freebairn and Rausser (1975), Arzac and Wilkinson (1979), Brandt et al., (1985), Stillman (1985), Holt and Johnson (1986), and Skold and Holt (1988). For example, in Arzac and Wilkinson (1979), beef supply is hypothesized to be determined by the inventory of beef cows calving which in turn is dependent on relative profitability expectations. The size of the calf crop determines total beef slaughter. This general structure is continued to be replicated later in many livestock econometric models.

Among the simultaneous equations models, Langemeier and Thompson (1967) considered beef cow breeding herd inventory as predetermined outside the system, while Folwell and Shapouri (1977) treated these inventories as endogenous and functions of expected prices. Ospina and Shumway (1979) estimated demand, supply and inventories of beef disaggregated based on quality. Explicit account of different effects of current and expected price was taken into this model. Competition in the resource use (especially feed grains) warranted the incorporation of hog and broiler subsectors in their beef model. Expected prices were generated by a polynomial distributed lag model of annual prices prior to the year of decision making.

Johnson and MacAulay (1982) used the information on biological relationships to obtain restrictions on the parameter estimates in the supply structure of their quarterly beef model. This approach has been subsequently used by Okyere (1982) and, Okyere and Johnson (1987) for beef, by Chavas and Johnson (1982) for poultry, and by Blanton (1983) and Oleson (1987) for pork. Chavas and Klemme (1986) demonstrate through their analysis of investment behavior in the U.S. dairy industry that the biological restrictions underlying the milk production can also be imposed in the specification of functional form. In this method, the biological restrictions remain intact and allow more producer behavioral discretion unlike the method of direct parameter restrictions.

The list of economic variables included in the livestock supply equations has extended beyond input and output prices. Measures of relative profitability in competing enterprises have been included to reflect the opportunity cost in production. For example, MacAulay (1978) included beef feeding margin while, among others, Freebairn and Rausser (1975) and Arzac and Wilkinson (1979) included producer price of cattle in their sow inventory equation. Heien (1976) developed an econometric model of the U.S. poultry industry using annual data over the period 1950-1969.

In his model, for a sample, broiler supply equation includes variables like broiler wholesale price, the feed cost, wage rate for the broiler industry, an industry capacity measure, and a time trend variable. The broiler and turkey model used by USDA and documented by Yanagida and Conway (1979) had its origin in Heien (1976). In this model, chicken production is specified as a function of eggs placed for hatching, wholesale broiler price deflated by feed cost and a trend factor. Turkey production is estimated based on the farm price of turkey, feed cost, an index of fuel costs and a trend variable. More recently, the USDA has developed and documented a model of the livestock sector (Stillman, 1985; Westcott and Hull, 1985). Their model includes both biological and behavioral equations for beef, pork, broiler, and turkey production and their prices.

The general structure of the feed demand component of livestock model reflects usually, the theoretical framework of derived input demand functions. A good review of the extensive literature dealing with the derived input demand functions in livestock production is found in Womack (1976). As such, these feed demands depend on their own prices, prices of alternative feeds, prices of the final livestock products and the number of grain consuming animal units (GCAU). For given prices of livestock products, the feed grain demand system is recursive to the livestock supply system. Therefore, the GCAU taken as explanatory variables in the feed grain demand functions should be obtained from the simultaneous solutions of the livestock supply system. Most of the livestock supply studies in the past adopted this approach to estimate the feed demand component of livestock model (CARD/FAPRI model in Skold et al., 1988; CARD/BLS model in Abkin, 1985). However, the specification of these input demand equations and their linkages to the output equations in these models do not follow from any suitable theoretical model of producer optimization behavior. Despite the fact that the GCAU is a useful statistic which compares the production of various livestock products in terms of the feed consumption of an average milk cow, there are some obvious drawbacks in using such a highly aggregated system. The geographical dependency of feeding practices and livestock production necessitates the nature of the GCAU to be different from one feed grain to another. Because of the widely differing feeding regimes used in different regions, an average system like GCAU can yield only approximate answers.

Among much debated issues in livestock production analyses are the sign and magnitude of the short-run and long run elasticity estimates. Empirical results obtained from econometric estimation of aggregate livestock supply functions have varied widely among studies. In particular, wide variations in long run elasticity estimates are a common occurrence. For example, Wipf and Houck (1967), Hammond (1974), Chen et al. (1972), and Hutton and Helmberger (1982) report the short-run elasticity of milk supply to be very small (between 0.07 and 0.16). However, the long run elasticity estimates for milk supply have varied from a low of 0.14 (Hammond, 1974) to a high of 2.53 (Chen et al., 1972). As for beef supply, elasticity estimates range from negative short-run elasticities (-0.01 by Tryfos, 1974 to -0.17 by Reutlinger, 1966) and positive long run elasticities (0.037 by Cromarty (1959) to 0.16 by Skold et al. (1988)). To quote Knight (1961), for example:

> ... research workers have probably had more difficulty deriving meaningful and realistic supply price elasticities for beef than for any other commodity.

highlights the difficulties involved in coming up with realistic or common empirical estimates of elasticities in livestock models. Differences in elasticity estimates from different studies arise due to several reasons. Differences in capturing the dynamics inherent in livestock production is an important reason for the differences in elasticity estimates.

These wide variations in the elasticity estimates indicate that additional research on the dynamics of livestock production response is necessary. Failure to explicitly and correctly incorporate the dynamic adjustments between outputs and inputs through the production technology specification makes it harder to interpret the short-run and long run aspects of elasticities. An attempt to overcome such a pitfall in modeling would help better understand the speed and magnitude of economic adjustments in livestock production and would enable in better assessment of long run production impacts of alternative livestock policy options.

Several studies have demonstrated the usefulness of multioutput supply response modeling framework in agriculture (Shumway, 1983; McKay et al., 1983; Lopez, 1984; Shumway and Alexander, 1988). However most models of supply response in livestock production focus on aggregate supply response for a single commodity in a partial equilibrium framework ignoring the interdependencies between various outputs, breeding herd inventories and other inputs within this sector. For example, Dahlgran (1980, 1985), Chavas and Klemme (1986), and Howard and Shumway (1988) modeled U.S. dairy industry in isolation while Chavas et al. (1985) studied swine production and Chavas and Johnson (1981) studied egg production. Only a very few researchers attempted to model the whole livestock sector together with feed demand as a complete system (Stillman, 1985; CARD/FAPRI model in Skold et al., 1988; CARD/BLS model in Abkin, 1985). However, a critical examination of these models would shed some light on the weak theoretical foundation upon which the behavioral restrictions rest. In these models, neither the structure of various equations nor the linkages between them follow any rigorous theoretical treatment of producers' optimizing behavior. Thus any future research efforts should recognize the importance of the analysis of price and output-input adjustments at sectoral level while adhering to the underlying economic theory of producer behavior.

In the past, modeling of livestock production response has relied heavily on static tools with *ad hoc* specification of behavioral equations. However, livestock production processes are neither instantaneous nor static. For instance, there are some sector-specific capital inputs that do not adjust instantaneously in a short period. Besides livestock production is inherently dynamic because of biological lag associated with the growth process. Such rich information about the dynamics of growth must somehow be accounted for in a theoretically consistent and empirically tractable fashion in any livestock production model. This is what makes modeling livestock sector rather more challenging than modeling crop sector (except, of course, few types of perennial trees!).

Modeling of production dynamics can be done in several ways. One approach, as discussed by Dillon (1977), specifies production response as function of time and total input used during the response period. The most common means of incorporating dynamic elements in production and factor demand analysis has been through univariate-flexible accelerator (partial adjustment) mechanism (Lucas, 1967; Treadway, 1969; Mortensen, 1973). The major limitation of this approach is that it posits constant adjustment rate for a single quasi-fixed factor independent of other similar inputs. Hence, it will not be useful as such for livestock supply modeling where there are more than one output and one fixed input to consider.

Many of the past livestock supply studies, whether single commodity or multiple commodities, including the more recent ones like Dahlgran (1985), LaFrance and de Gorter (1985), and Chavas and Klemme (1986) use some sort of lag structure to acknowledge the fact that the decisions made today about breeding and culling take time before their impact is felt. However, these models incorporate dynamics in a largely *ad hoc* manner. Nerlove (1972) laments that the application of distributed lag models in empirical economic studies

> ... is astounding but what is more remarkable is the virtual lack of theoretical justification for the lag structure superimposed on basically static models.

A rigorous way of improving the theoretical strength of supply dynamics is the explicit treatment of the optimization process implicit in the firms' supply decisions, i.e., by incorporating behavioral restrictions implied by intertemporal optimization of the firm. Dynamic models that are consistent with the theory of firm have been derived from applications of optimal control theory but have not been used widely in livestock supply modeling. Primal and dual models can be derived from an intertemporal value function in the form of a Hamilton-Jacobi equation. The behavioral equations may be obtained via a primal approach using first order Euler equations or via dual approach by applying envelope theorem to the value function. The primal approach was developed by Treadway (1970) and has since been used by Berndt et al. (1981a, 1981b) to model U.S. manufacturing sector and by Lopez (1985) to model Canadian food processing industry. This primal approach is limited to modeling only one quasi-fixed input or with the assumption of independent adjustment between two or more quasi-fixed inputs. Hence, this approach will not be suitable for the present purpose of modeling U.S. livestock production and factor demand with more than one quasi-fixed factor (like various capital services, and breeding herd inventories) whose optimal adjustments are generally interdependent.

A large body of empirical studies of factor demand and production (Christensen et al., 1973; Berndt and Christensen, 1973; Fuss and McFadden, 1978) is based on the assumption that the firms can adjust all factors instantaneously to changing prices. Multiproduct firm utilizes both variable inputs and quasi-fixed inputs to produce many outputs. Quantities of variable inputs can be adjusted completely within the current time period. However, full adjustment of quasi-fixed inputs can be hypothesized to be accomplished only by incurring some cost, either directly or in the form of foregone output. There is a sizable literature justifying the existence of such adjustment costs (external as well as internal) for firms (Penrose, 1959; Eisner and Strotz, 1963; Lucas, 1967). Furthermore, Maddox (1960), Baumgartner (1965), and Gallaway (1967) document the relevance of these costs for agricultural enterprises. The adjustment cost hypothesis has profound implications to the distributed lag behavioral response of agricultural production to changing economic conditions. Here, sources of supply dynamics are not explicitly modeled. But the sluggish response of quasi-fixed inputs (due to the presence of some adjustment cost) to changing market prices gives rise to the observed distributed lag pattern. The distinction between short-run and long run behavioral responses of producers can be handled in a theoretically consistent fashion using this adjustment cost hypothesis (Vasavada and Chambers, 1986).

In multiproduct framework, the task of obtaining a closed form solution to input demands and output supplies after the specification of a relatively general technology (say, in the form of a 'well behaved' transformation function), in many practical instances, is insurmountable. Thus, for analyzing multiple output and input relations in an econometrically amenable form, specification of a differentiable profit (value) function and application of multiproduct version of *Hotelling's lemma* offers an attractive alternative. Note that this dual approach does not require information on output-specific input use for estimation (Shumway, 1983; Lopez, 1984). Information on sectoral aggregate input use across all outputs is all that is needed. This aspect of duality is particularly useful for the present purpose of modeling livestock production and factor demands, because, in the U.S., livestock product-specific feed and other factor use information is not readily available for all outputs considered in the present study. Also, since duality yields explicit reduced forms with prices as independent variables, more simple econometric estimation techniques could be employed.

The first application of duality to the agricultural production may be found in Lau and Yotopolous (1972). Since then, the number of applications of duality to agriculture has steadily increased due to the ease in its empirical implementation. However, static duality approach will not be enough for the present purpose, since a good livestock supply model must account for the dynamic nature inherent in production and factor adjustments (including herd inventories). In static models, duality is a convenience. For empirical intertemporal optimization problems, explicit solutions of differential equations system derived from the primal technology function are too complicated. This renders intertemporal or dynamic duality as an indispensable tool for empirical work in livestock production and factor demand analysis.

The intertemporal dual approach was initiated by McLaren and Cooper (1980) and formalized by Epstein (1981). Since then there is a growing interest in the empirical application of this approach as evidenced by the growing literature in this area. For instance, Epstein and Denny (1980) employed this technique to study the U.S. manufacturing sector while Taylor and Monson (1985) and Vasavada and Chambers (1986) used it to analyze the U.S. agriculture. Howard and Shumway (1988) is the only empirical application of this approach to livestock sector per se. However, they model the U.S. dairy industry in isolation.

Objectives

Dynamic dual approach is used in the present study since livestock production is characterized by the multi-output and multiple interdependent variable and quasi-fixed inputs. The model used in the present study is a structural one with the structure derived explicitly from relevant economic theory of producer behavior. Restrictions placed on the model are not *ad hoc*; rather they are implied by the underlying optimization behavior of producers. Several outputs, variable inputs and quasi-fixed inputs are considered as an interdependent system at sectoral aggregate level. Specifically, beef, milk, pork, chicken, eggs, turkey, sheep and lambs, and wool are the various outputs considered while labor, operating capital, and feed (grain feed, protein feed, and hay) are the variable inputs considered. Capital services (buildings, machinery, and equipments) and various inventories of breeding animal stocks are treated as quasi-fixed inputs. The purpose of the study is to examine the dynamic structure of the U.S. livestock sector. Supply response and factor adjustments for varying economic conditions will be examined. The specific objectives of the present study are:

1. To develop and estimate a multiproduct production and factor demand model for the U.S. livestock sector,
2. To evaluate the dynamics of production and factor adjustments in the context of short-run and long run,
3. To simulate the production and factor demand responses for changes in relevant exogenous economic stimuli, and
4. To assess the implications of the present modeling approach as well as of the policy simulations from the empirical results.

Organization of the Study

The present study is organized under six chapters. Chapter 1 identifies the problem that is investigated and lays out the objectives of the study. Chapter 2 provides some background information about the U.S. livestock sector and government programs that affect it. Chapter 3 provides a comprehensive overview of the theory of dynamic duality and a discussion about the proposed theoretical model and various issues in implementing this methodology to the present study. In Chapter 4, development of various data series used and their sources are presented. In addition, the econometric estimation procedure adopted for the empirical model is outlined in this chapter. Chapter 5 reports the empirical results and their implications. Model validation and results of policy simulation exercises are also discussed in this chapter. Finally, Chapter 6 contains a summary of the findings of the study together with suggestions for future research in this area.

CHAPTER 2. U.S. LIVESTOCK SECTOR: SOME BACKGROUND INFORMATION

A clear understanding of the structure and characteristics of the livestock sector is essential before one embarks on the mission of modeling for policy evaluation. Thus, a brief description of the main characteristics of various industries within the U.S. livestock sector followed by a description of relevant government programs that affect this sector is presented in this chapter.

There are four main program types that are generally used in the U.S. farm policy: the production subsidy or deficiency payments, market-floor price support (dairy), production control (tobacco, and peanuts) and import restrictions (beef, and sugar). The programs relevant to the livestock sector are listed in Table 2.1.

Dairy

The current state of the dairy industry is one of over production and escalating government costs. In 1983, nearly 139 billion pounds of milk was produced of which only about 122 billion pounds found its way to commercial markets. The excess production (about 12 percent of the total production) was purchased by the government in order to assure the farmers the announced dairy support price. The cost of this program to the government was $2.35 billion in 1986 as compared to $1.03 billion in

Table 2.1: Major livestock and feed grains programs in the U.S. (1985-86)

Commodity	Program	Farm value of production \$ Billion	Government expenses \$ Billion
Dairy	Price supports	18.38	2.35
	Import controls		
	Marketing orders		
	Whole-herd buy-out		
Cattle	Import restraints	29.05	0.0
Wool and mohair	Price support	0.10	0.10
Hogs	No programs	9.79	0.00
Poultry	No programs	0.01	0.00
Feed grains	Price supports	25.40	12.28
	Deficiency payments		
	Acreage diversion		
	Storage subsidies		
Soybeans	Price supports	10.57	1.60
Hay	No program	9.44	0.00

1980. The excess supply problem is the result of the price support program that had set the support price significantly above the market clearing price. The 1985 farm legislation provided some features related to lowering the support price, producer assessments and a whole-herd buy-out to correct this excess supply problem.

The U.S. dairy industry is a domestic market largely cut off from trade. Import levels have been less than 2 percent of production since 1975. Exports, mostly subsidized, constituted between 2 and 3 percent of domestic production. Dairy products exports are mainly of government controlled Commodity Credit Corporation's (CCC) stocks which constitute about 3 percent of annual U.S. milk production. The legislated import quotas limit the import of dairy products to an equivalent of 2 percent of U.S. production. Although the exported and imported products are different, the U.S. in world dairy market can be considered essentially a nontrading entity with the present policy regime. However, if the U.S. is to dispense with its price support without relaxing the import barrier, the domestic farm price will fall but still will be considerably higher than the world price. Currently the world milk price averages about one-third of the U.S. support price (USDA, 1984a).

The milk producers receive a market support price guarantee from the government through the operations of its CCC. Unlike for the grain crops, no deficiency payments are made to the milk producers. Instead, the CCC buys cheese, butter, and non-fat dry milk at prices calibrated to generate the legislated support price to the producers. Consumers pay the raw-material-price that farmers receive. The legislated import quotas prevent the CCC from supporting the generally lower world price. Thus, the domestic prices for the dairy products are artificially held high. Excess supply at this higher support price has been about 10 percent of annual production during the 1980s. This surplus is purchased by the CCC, stored and mostly distributed as food aid domestically or abroad. Net government expenditure for this intervention is about $1.6 billion per year.

The dairy industry is also aided by *marketing orders*, which are essentially arrangements under which milk marketing cooperatives can charge higher prices for fluid milk (Class I) for drinking, as compared to manufactured milk products. The role of marketing orders in regulating milk markets has increased significantly in the past two decades. Marketing orders enforce classified pricing whereby milk is priced according to its final use. Producers do not receive the price the handlers pay, but a *blend price* that is the average of the Class I and manufacturing prices weighted by the proportion of milk used for each purpose throughout the marketing order.

Within each order, a minimum price is set at which handlers can buy milk for fluid purposes. Because the support price has generated greater supply than commercial markets demand, the market price for manufacturing milk has generally been at or near the government support price. The blend price that producers receive is thus a combination of the Class I price and the national support price. Producers gain about \$500 million annually from selling fluid milk at prices above the stipulated support price, thanks to the marketing order. However, the dead weight loss associated with the marketing order is estimated to be small because of inelastic demand for fluid milk and a small size of price premium - about 8 percent over the manufacturing milk price (Gardner, 1985). Note that this marketing order does not restrict production directly so that free entry will tend to eliminate any rent that may be created.

The whole-herd buy-out is another one-time provision of the 1985 farm bill that was intended to reduce the milk cows inventory in a relatively short period of time. Nearly 10.3 percent of the total milk cows were removed under this program with government payments totaling \$489 million.

Beef

During the past several years the U.S. beef industry has experienced continuing structural change. The size of the production enterprises within the industry has expanded while the total number of producers has decreased. Through improved production practices and technological innovation, beef producers have increased calving rates, reduced death loss, increased the rate of weight gain, and increased feed efficiency. These improvements are attributed to improved breeding techniques, disease control, and increased use of growth-stimulating hormones and feed additives.

Changes in regionality of production have also occurred with production and marketing shifting from the Corn Belt- Lake State regions to the Central plains during the last two decades (Van Arsdall and Nelson, 1984).

Beef industry has no price support system. However, imports are restricted by means of a tariff of 2 cents per pound and by voluntary restraint agreements under which beef exporting countries agree to hold down exports if projected imports exceed certain trigger level established by the administration. The estimated effects of these restrictions are small. Besides, the imported beef is generally *lean cow beef* which is less important than grain-fed beef in U.S. beef consumption. Nevertheless, Gardner (1985) reports that these import restrictions transfer about $500 million annually from the consumers to the producers. Exporting countries who have access to the U.S. beef market through the agreement also gain while other beef exporting countries lose due to lower world price.

Pork

The U.S. hog industry has experienced dramatic structural change, attained more production per sow, more production per unit housing, and lower feed costs (Van Arsdall and Nelson, 1984). Pork production also has become less seasonal with the adoption of large capital-intensive confinement operations. Pork production remains regionally concentrated with almost 70 percent of the total production occurring in the Corn Belt states. Production continues to be dominated by farrow-to-finish operations with the producers retaining control over the entire production phase: from breeding to birth to slaughter. Pork industry is relatively free from government regulation over production.

Poultry

Among agricultural industries, probably none has changed more rapidly in terms of production location, vertical integration, and technology in recent decades than the poultry industry. This industry has represented roughly 10 percent of total cash farm income during the past several decades. Production of eggs, broilers, and turkeys has increased while that of mature chicken (residual from egg and broiler production) declined substantially in the last few decades (USDAAS). Advances in poultry production technology have been substantial in the past. The increased productivity as a result of breeding, feeding efficiency, disease control, and management enabled the firms to gradually bring various phases of poultry production under one vertically coordinated management. This vertical integration allows for the analysis of poultry production as part of a single process, unlike beef and pork production.

Sheep, Wool, and Mohair

Annual U.S. wool production is equivalent to only about one-tenth of one percent of the value of principal crops produced in the U.S. The sheep marketings are about the same fraction of the value of the total livestock marketings. The value of mohair produced is but only a third of wool's value. However, the significance of these fibers is substantial in production areas, particularly in parts of Texas and the Rocky Mountain States where crops would fare poorly or cannot be grown. The recent performance of the wool market and the experience with the Agricultural and Food Act of 1981 have raised various issues to consider when assessing policies for the future. Some such issues are: a) should there be a wool and mohair program at all? and b) have the program costs exceeded acceptable limits? Support functions purely as an income supplement to producers; legislation does not require production cutbacks in return for support payments. Legislation has set support levels for wool consistently above world prices and attempted unsuccessfully to revitalize the declining wool industry. The outcome has been predictable, namely, rising imports and government costs.

Today's wool and mohair price support programs are the consequence of several laws passed between 1938 and 1981. Most significant was the National Wool Act of 1954, which created the program provisions that are essentially in effect today. The significant feature of the program for the producers was that *direct payments* were authorized as a method of supporting incomes, and since 1955 it has been the only method used. The method of computing the wool and mohair payments differs from that used for major crops where the producers receive a fixed payment per unit of production. The wool and mohair payment per unit of production increases as the value per unit of the output increases. This payment, called the "incentive payment," is supposed to encourage the production of higher quality (hence higher market value) wool. The payment rate is based on the percentage needed to bring the national average market price received by the producers to the support price. Thus, the higher the price a producer receives for his wool, the higher is the per pound incentive payment.

Currently, a major concern is the escalation of the support prices. In 1983, the support price of $1.53 per pound of shorn wool was $2\frac{1}{2}$ times the average market price. Government payments were at a record $116 million for wool and $6.4 million for mohair. Besides, the wool program payments are not subject to a payment limit unlike the payments under crop programs.

The program benefits accrue almost entirely to the producer mainly because the demand is more responsive to the changing prices than the supply is. In addition, the small size of the U.S. wool market in relation to the world market and the substantial volume of U.S. wool imports suggest that the domestic wool price is more related to the world price than to the incentive payment. Thus, the consumers benefit very little from this program. On the other hand, the tariffs charged on imported raw wool (about 10 cents per pound) and wool textiles provide a very significant level of protection for the domestic wool industry, raise domestic prices, and reduce consumer welfare. Such tariffs raise revenue ($241 million in 1983) that more than offsets the government's program expenditures (USDA, 1984b).

Besides the above mentioned government programs that are specific to each industry, there are other government measures like crop commodity programs (for feed grains, in particular), subsidized credit and insurance (e.g., disaster payment and emergency loan program), tax-shelter farming, and other subsidies (like the federal irrigation project subsidies) also benefit indirectly the livestock sector because of interdependencies between various sectors of the economy.

CHAPTER 3. DYNAMIC DUALITY: THEORY AND PRACTICE

Some basic results of duality theory which are instrumental in the specification and estimation of multioutput-multiinput model of the U.S. livestock sector considered in the present study are presented here. Fuss and McFadden (1978), Lau (1978), Epstein (1981), and McLaren and Cooper (1980) are the basic references for the material in this chapter. Various issues like choice of functional form, data aggregation, Nonjointness in production, and expectation are discussed as they relate to the empirical implementation of the proposed model of this study. Finally, detailed derivation of the system of equations for output supply, variable inputs demand, and quasi-fixed inputs investment demand based on a normalized quadratic value function as a dual representation of production technology is presented.

The production function specification and estimation in livestock modeling has received considerable attention in the literature. Dillon (1977) provides a concise summary and bibliography on this subject. As a method for estimating supply response, the direct production function approach has considerable limitations. As Kehrberg (1961) pointed out, only in trivial cases where products are either completely independent in terms of factor competition or where joint products are produced by the multi-product firm in fixed proportions, might single commodity production functions be accepted as a sound theoretical basis for estimating supply response. Another difficulty with the direct production function approach is that of simultaneous bias. It is accepted that in reality, the levels of inputs and outputs are jointly and simultaneously determined in the light of exogenous economic determinants like pıices. Hence, to treat the the levels of inputs as exogenous determinants of output is not wholly appropriate. Estimating dual profit function overcomes this problem and this is one of the justifications explicitly given by Lau and Yotopolous (1972) for their choice of profit function approach over primal representation of production to estimate supply response.

The conceptual basis of duality theory rests on the notion that there is an *alternate, but equivalent* way to represent production technology. This proves to be particularly handy when there are limitations imposed jointly by data availability and parametric specifications. The first modern rigorous development of duality theory is due to Shephard (1953). The first application of duality to agricultural production was by Lau and Yotopolous (1972).

According to the principles of duality (Fuss and McFadden, 1978; Blackorby et al., 1978), there is a direct equivalence between the production and cost, and production and profit function and any one of these three functions could be econometrically estimated to derive supply response parameters.

A reciprocal correspondence exists between the production (transformation) function and profit function, such that with profit maximization, there exists a dual relationship between a regular production function and a profit function. Lau and Yotopolous (1972) state that

> McFadden has shown that there exists a one-to-one correspondence between the set of concave production functions and the set of convex profit

> functions. Every concave production has a dual which is a convex profit function and vice-versa. Hence, without loss of generality one can consider only profit functions in the empirical analysis of profit maximizing, price taking firms.

It is this relationship, that all dual functions contain the same basic information, which creates the possibility that the output supply and input demand functions can be derived from any one of the dual functions.

Duality

Static Duality

The principal advantage of specifying dual profit function rather than its primal production (transformation) function in empirical work is the simple relation between the profit function and the corresponding input demand and output supply functions known as the *Hotelling's lemma.* This relationship allows the derivation of input demand and output supply equations from the knowledge of the profit function alone by simple differentiation.

Consider a multiproduct firm producing m outputs using n variable inputs and k (quasi) fixed inputs. Given a vector of output prices p, a vector of input prices w, and a production possibilities set T, then the firm's variable or restricted profit function is defined by:

$$\Pi(p,w,Z,\dot{Z}) = \max_{Y,X} \left\{p'Y - w'X \mid X,Y \in T(Y,X,Z,\dot{Z})\right\} \tag{3.1}$$

where

- Π is variable profit (gross revenue less variable cost),
- T is production possibilities set
- p is $m \times 1$ vector of output prices,
- w is $n \times 1$ vector of variable input prices,
- Y is $m \times 1$ vector of outputs, and
- X is $n \times 1$ vector of variable inputs.
- Z is $k \times 1$ vector of (quasi) fixed inputs,
- $\dot{Z}$ is $k \times 1$ vector of new net investments,

The profit function, Π, is assumed to satisfy the following regularity conditions:

1. Π is a proper twice continuously differentiable function of $(p, w, Z, \dot{Z})$
2. Π is nondecreasing in p,
3. Π is nonincreasing in w,
4. Π is nondecreasing in Z,
5. Π is convex in (p, w)
6. Π is concave in $(Z, \dot{Z})$
7. $\Pi(p, w, Z, 0) \geq 0$

The implications of these properties of Π are outlined in more detail in Diewert (1973, 1974), and Lau (1978). However, the appearance of $\dot{Z}$ in the production possibilities set T warrants further justification. One can posit that the firm has to incur some external cost of adjustment from the idea that a *premium* must be paid to acquire new investment goods. However, the fact that $\dot{Z}$ is included here in T is to imply that a quasi-fixed factor can be changed only by incurring some internal adjustment costs. This notion of quasi-fixity is due to Treadway (1970, 1971). Brechling (1975) defines such internal costs as being equivalent to the proposition that the inputs used by the firm at one point in time are at least 'partially' produced by the firm at some earlier date. Morrison (1982) adds that, since production of output and changing input levels are joint processes, a more rapid change in input levels can be obtained only at the expense of output (if the resources are given), or by increasing resources (if output is given), each resulting in increased internal costs of adjustment. These costs increase with increase in the amount of adjustment, that is, the cost of adjustment is convex.

The assumption of convexity of adjustment costs merits further explanation. Specifically, the firm will adjust its capital stocks instantaneously to changes in market conditions if there are no adjustment costs or if costs of adjustments are linear or concave. However, the rationale behind the assumption of convexity of adjustment costs is that it becomes more and more expensive to adjust things quickly than slowly. In livestock production, the breeding herd adjustments to given economic stimuli do take time due to the underlying biological lags involved in the growth process. Besides, such adjustments do involve some kind of internal costs since the producer has to divert some limited resources (like his/her managerial skill) away from production to this adjustment process itself. Thus, the assumption of convex internal cost seems reasonable.

Hotelling's lemma: If the profit function, Π, satisfies the regularity conditions listed earlier, then the profit maximizing output supply and input demand (for given Z and $\dot{Z}$) are obtained by simply differentiating Π with respect to the respective prices to yield the following system of equations:

$$\Pi_{p_i} = Y_i(p, w, Z, \dot{Z}) \qquad \forall i = 1 \ldots m \tag{3.2}$$

$$-\Pi_{w_j} = X_j(p, w, Z, \dot{Z}) \qquad \forall j = 1 \ldots n \tag{3.3}$$

where $Y_i(\cdot)$ is the profit maximizing output of i^{th} commodity, and X_j is the profit maximizing j^{th} variable input demand for given $(Z, \dot{Z})$. These equations form a system that can be readily estimated using suitable econometric tools. Note that Y_i and X_j are determined based on the given levels of Z. This does not explain how the optimal levels of Z are determined or how these quasi-fixed factors evolve over time due to changes in $\dot{Z}$. Thus, static duality is not sufficient for modeling the dynamics of livestock production and breeding herd adjustments.

Dynamic Duality

There is no doubt that dynamic analysis of livestock product supply responses is exceptionally complex. The reasons for this are that for 'a given animal at a given time may be viewed as (a) a finished good, (b) a good in production process, or (c) a piece of fixed capital' (Hildreth and Jarret, 1955). These characteristics indicate the need for simultaneous approach to explaining outputs, inputs, and herd inventories. Watson (1970) has gone so far as to argue that, in livestock models, the complexities arising from the underlying investment decisions are such that in time-series regression analysis no satisfactory explanation of supply in terms of prices alone is likely to be possible. In his view, this arises because the relationship between any exogenous price and the desired and actual levels of inventory are not likely to be constant. This emphasizes the fundamental connection between the incorporation of price expectations into supply models, the lagged role of investment decisions upon supply, and the consequent dynamic nature of supply responses to price. Models which simply adopt an *ad hoc* specification in which price expectations are assumed to directly influence supply are short-cutting the need to specify the investment behavior underlying the supply response.

While in principle the duality relationships need not be restricted to static optimization problems alone, in practice little effort seems to have been made to exploit these relationships in a dynamic modeling context. One approach to intertemporal production theory is to formulate production relations in terms of all observable and measured inputs and outputs, with goods being distinguished as to both physical type and time period in use (Hicks, 1946; Malinvaude, 1953). This type of general formulation requires that an unreasonably large number of parameters be estimated. Therefore, a more pragmatic supply response modeling approach, like dynamic duality, must be adopted in order to make it empirically tractable, yet, capturing the essentials of the dynamics of the production process.

Dynamic or intertemporal duality theory establishes the relationships between the production function, restricted profit function and intertemporal value function of the firm. The intertemporal analogue of *Hotelling's lemma* (McLaren and Cooper, 1980) can then be employed to derive the output supply, variable input demand, and optimal investment in quasi-fixed factors from the knowledge of firm's intertemporal value function alone. A complete knowledge of the value function is all that is necessary in order to be able to infer a complete characterization of these factor demand and output supply functions that are consistent with the adjustment cost theory of firm. Recall that the adjustment cost hypothesis implies that the intertemporal link in the firm's technology is due to the fact that the levels of some quasi-fixed factor stocks may be changed only subject to increasing marginal cost of adjustment.

Consider a profit maximizing, competitive firm with a restricted profit function Π as defined in equation 3.1. The firm is assumed to possess an initial endowment of k quasi-fixed factors, Z_O and the ability to buy new capital goods I at given market prices. The firm, at any given point in time $t = 0$ (called the base period), is assumed to solve the following infinite horizon problem:

$$V(p, w, c, Z_O) = \max_{X, I} \int_0^\infty e^{-rt}\{pf(X, Z, \dot{Z}) - w'X - c'Z\}\, dt \qquad (3.4)$$

subject to

$$\dot{Z} = I - \delta Z$$

$$X, Z \geq 0$$

$$Z(0) = Z_O$$

where

- V is 'the present value function' that is central to dynamic duality,
- f is a 'well behaved' production function (i.e., twice continuously differentiable, concave over the relevant range of production), $f_X, f_Z > 0$ and $f_{\dot{Z}} < 0$ implying convex adjustment costs,

- c is the vector of rental prices for the quasi-fixed factors,
- r is a constant real discount rate,
- δ is a $k \times k$ diagonal matrix of constant depreciation rates such that δ_k is the relevant rate for the k^{th} quasi-fixed factor.

All variables, except Z_O, are implicit functions of time, so time subscript t is dropped to minimize notational clutter.

The prices denote actual market prices at $t = 0$, which are expected to persist indefinitely. This is to say that current prices contain all relevant information about future prices. The implications of this crucial assumption is discussed later in this chapter. As the base period changes, new market prices are observed, price expectations and production decisions of previous period are revised; thus, only that part of the plan corresponding to $t = 0$ is implemented in general.

A discrete time, fixed planning horizon formulation may be more natural for the problem on hand. However, if the investment rule is linear as above, the two formulations are equal. The investment rule from the continuous time model is first order Taylor approximation of the investment rule from the discrete framework (Karp and Shumway, 1984). However, even if the investment rules are the same with the discrete and continuous time formulation, the demand system for variable inputs will be different due to the difference in discounting. A linear investment rule is preferred for reasons of consistent aggregation which is imperative for present purpose because of the high degree of aggregation in the available data for estimation. Theoretical models are often specified in continuous time for its various advantages (Koopmans, 1950). However, data available to implement these models empirically are almost always discrete, often over intervals such as a quarter or a year. This may bring in a misspecification problem in terms of adapting the continuous theoretical model to discrete time data. Reliability of such applications is reduced as the discrete time intervals become wider. However, the magnitude of this problem cannot be directly inferred, and therefore assumed away to be small (Morrison, 1982). Continuous time, infinite horizon formulation is used here to keep the exposition tractable. Qualitative aspects of the discussion and conclusions basically do not change in either formulation.

The derivation of optimality conditions for the problem in Equation 3.4 is based on the *Maximum Principle* (Arrow and Kurz, 1970). A simpler form of the optimality conditions is available for infinite horizon autonomous problem like Equation 3.4 (Kamien and Schwartz, 1981, p. 241). The current value Hamiltonian for the present problem is:

$$H(p, w, c, \lambda) = max\{p' f(X, Z, \dot{Z}) - w'X - c'Z + \lambda'\dot{Z}\} \tag{3.5}$$

The optimality conditions, following Bellman's *principle of optimality*, are :

$$\begin{aligned} \dot{Z} &= H_{\lambda} \\ \dot{\lambda} &= r\lambda - H_{Z} \\ \lambda &= c - f_{\dot{Z}} \end{aligned} \tag{3.6}$$

Assuming one can represent the production technology via a 'well behaved' production function ,$f(\cdot)$, one can solve these optimality conditions for X^*, $\dot{Z}^*$. However, arriving at a closed form solution to the primal problem in practice is rather tedious. One has to seek the relative simplicity offered by the dual approach. Varying the initial condition Z_O, one can synthesize optimal investment function $\dot{Z}^* = \Phi(p, w, c, Z)$ and redefine the optimal value function V in terms of Φ as:

$$V(p, w, c, r, Z) = \int_0^\infty e^{-rt}\{pf(X, Z^*, \dot{Z}^*) - w'X - c'\dot{Z}^*\}\, dt \qquad (3.7)$$

where Z in V is now any arbitrary initial condition. Equation 3.7 can now be differentiated along the optimal path to yield the following Hamilton-Jacobi equation (Arrow and Kurz, 1970):

$$rV(\cdot) = \ max \ \{p'f(X, Z, \dot{Z}) - w'X - c'Z + V_Z\dot{Z}\} \qquad (3.8)$$

where V_Z is the (current value) shadow price of the quasi-fixed factor. Equation 3.8 is the basic ordinary differential equation obeyed by the optimal current value function V associated with the problem in Equation 3.4.

The Hamilton-Jacobi equation allows us to transform the dynamic problem in Equation 3.4 into a more manageable static form. Equation 3.8 simply states that the value function is the discounted present value of the current profit plus the marginal value of optimal net investment. The optimal value function V derives its properties both from the assumed conditions on Π or on f and the optimality conditions for the problem in Equation 3.4. The regularity conditions on f are fully manifested in Π and hence in V. Epstein (1981) has shown the conditions under which a dynamic dual correspondence exists between f and V. Specifically, V is assumed to possess the following properties (McLaren and Cooper, 1980):

- V is a real valued function, twice continuously differentiable in its arguments,
- nondecreasing in p ,Z,

- nonincreasing in (w, c),
- convex in (p, w, c),
- concave in Z,
- $V_z \geq 0$ and V_{Zc} nonsingular.

Application of envelope theorem to Equation 3.8 provides a simpler way to derive a system of factor demand and output supply equations that are consistent with the intertemporal optimization framework. McLaren and Cooper (1980) state and prove a theorem that is central to intertemporal analogue of *Hotelling's lemma.* Given a V that satisfies the regularity conditions, a vector of positive prices (p, w, c), and r, a simple differentiation of Equation 3.8 with respect to various prices and some rearranging of terms give the following system of equations for output supply, variable input demand, and optimal quasi-fixed factor investment demand:

$$f(\cdot) = \vec{r}V_p - V_{pZ}\dot{Z} \tag{3.9}$$

$$X(\cdot) = -\vec{r}V_w + V_{Zw}\dot{Z} \tag{3.10}$$

$$\dot{Z}(\cdot) = V_{Zc}^{-1}(\vec{r}V_c + Z) \tag{3.11}$$

where the subscript(s) of V denote the partial differentiation of V with respect to the respective argument(s) and $\vec{r}$ is a diagonal matrix of appropriate dimension whose diagonal ellements are all a constant r.

Equations 3.9-3.11 provide a basis for empirical application of intertemporal duality. Closed-form expressions for optimal investment, variable factor demand, and output supply are expressed in terms of optimal value function alone. Thus, one can simply hypothesize a suitable functional form for V that satisfies the regularity conditions and apply the intertemporal analogue of Hotelling's lemma to obtain the system of equations (3.9-3.11) and estimate using appropriate econometric techniques. Note that to ensure the existence of a duality between f and Π or between f and V, it is essential either to impose *a priori* restrictions during estimation or to verify empirically if V satisfies its regularity conditions.

Issues in Implementation

Generalizations which would result in a richer set of choices available to the firm and reality of the models, would also increase the complexity of the empirical analysis. Therefore, in setting up a model for empirical implementation, a trade-off exists between realism and tractability. As in many previous empirical work, some crucial assumptions are made in the present study to apply the proposed theoretical model to the available data.

Flexible Functional Forms

The ultimate objective of the present study is to derive and estimate factor demand and output supply equations for the U.S. livestock sector. To implement the algorithm underlying the intertemporal duality, a parametric value function must be specified. In the standard duality theory much attention has been given to the so-called flexible functional forms which may provide second-order approximations to arbitrary functions (Diewert, 1974; Lau, 1974). Fuss and McFadden (1978) conclude that a necessary and sufficient condition for a functional form to reproduce comparative static effects (like output level, and various elasticities) at a point without imposing *a priori* restrictions is that it have $\frac{[(n+1)(n+2)]}{2}$ distinct parameters such as would be provided by a Taylor's expansion to second order. Generalized Leontief, Generalized McFadden, normalized quadratic, and translog are some examples of such flexible functional forms frequently employed in agricultural production analysis (Diewert, 1971; Sidhu and Baanante, 1981; Ray, 1982; McKay et al., 1983; Shumway, 1983; Lopez, 1984; Vasavada and Chambers, 1986; Diewert and Wales, 1987; Squires, 1987; Ball, 1988; Howard and Shumway, 1988).

Flexibility in the context of dynamic model is more stringent than flexibility within a corresponding static model of profit maximization. Epstein (1981) shows that while the dual Π in Equation 3.1 is regarded in standard static dual models as defining a flexible functional form, V exhibits various aspects of 'inflexibility'. A functional form for the value function V in Equation 3.8 is said to be flexible if the system of of factor demand and output supply functions as given by Equations 3.9-3.11 can provide a first order approximation at a point to a corresponding set of functions generated by an arbitrary value function that satisfies the conditions on V. It follows immediately that a functional form is flexible if and only if it can assume, at any point, any given set of theoretically consistent values for V, all first and second order derivatives of V and all first order derivatives of V_Z. Epstein (1981) discusses the properties of some functional forms for V in more detail.

It must be observed that the restriction on V_{Z^C} facilitates the determination of curvature properties of V. Unlike in static dual profit models, second order conditions on V alone are not generally sufficient to verify the necessary curvature properties of the underlying production technology. The reason for this is, that in dynamic setting, third order properties are of significance (Taylor and Monson, 1985). However, Epstein (1981) shows that if V_Z is linear in prices, then convexity of V in prices (p, w, c) is sufficient for the existence of desired curvature properties of the underlying production function. This, indeed, is the case for a V represented by generalized Leontief or quadratic function. In such a simplified representation, V_{Zc} simplifies to $(M - r)^{-1}$ where M is a matrix of constants known as the *coefficients of adjustment matrix*. Furthermore, Equation 3.11 for $\dot{Z}$ can be expressed as a multivariate flexible accelerator model:

$$\dot{Z} = M[Z - \bar{Z}(p, w, c)] \tag{3.12}$$

where $\bar{Z}$ is the long run desired level of Z consistent with intertemporal optimization framework of Equation 3.4. The M matrix enables the characterization of interdependency among various quasi-fixed inputs and their relative fixity in adjustment. To exploit these simplicities in interpreting empirical results, a quadratic value function is employed in the present study.

Aggregation

The model presented by Equation 3.4 is derived from the optimization analysis of a single economic entity based on firm-level theory. However, in reality, the behavior of the entire industry is modeled as a single representative firm using aggregate data. Such an approach is adopted due to the lack of firm-level data and the simplicity of aggregate models. The livestock sector consists of many price taking firms. The basic problem, then, is to determine the conditions under which a theoretically consistent aggregate optimal value function which only depends on the aggregate level of quasi-fixed factors and not on their distribution across firms can be hypothesized to exist. It is desirable that the aggregate value function satisfy the same theoretical restrictions as that of the firm's. Consistent linear aggregation, as applied by Vasavada and Chambers (1988) is one way to guarantee this. Such an aggregation rule requires that:

$$V(\cdot) = \sum_{S} V_S(Z_S) \tag{3.13}$$

such that

$$\sum_{S} Z_S = Z$$

where s indexes the firms, Z_S is the amount of quasi-fixed factor in use at s^{th} firm, and Z is the sectoral aggregate of the quasi-fixed factor available. Such a linear aggregation implies that the marginal effect of Z_S on the optimal value function of each firm (V_S) is identical and should equal to the marginal effect of aggregate Z on the aggregate V. In other words, V is affine in Z and $V_{ZZ} = 0$ (Epstein and Denny, 1980). Blackorby and Schworm (1982) suggest a less restrictive aggregation condition:

$$Z = \sum_{S} \Psi_S(Z_S) \tag{3.14}$$

This aggregation rule is not that useful in empirical analysis since it requires the knowledge of firm-specific Ψ_S functions. A detailed discussion on the aggregation conditions and their implications can be found in Chambers (1988). Little work has been done on aggregation in a dynamic model. What has been done so far indicates that some simplifying assumptions are inevitable to achieve empirical applicability of the otherwise complex theoretical model. A linear aggregation rule as implied by Equation 3.13 is used in the present study.

Ideally, more time-disaggregated data, say quarterly, would be preferable. Data limitations, however, prevent this since adequate information on labor, feed, and capital use, disaggregated more completely by time and product, is unavailable for the U.S. livestock sector. Besides, such disaggregated data, if and when available, tend to be simply interpolated yearly data. This would incorporate an additional source of error into an already econometrically complex model. It appears, therefore, that the annual level of data aggregation is justifiable for the present empirical model given the existing constraints on the data availability.

Separability and Nonjointness

Models that analyze multiple outputs typically specify transformation (or its dual) function which imposes *a priori* restrictions on the structure of production. Separability and Nonjointness in production are the most common functional structure assumptions imposed in empirical work. Separability assumption enables to justify multistage optimization which, in turn, permits consistent aggregation of all like-inputs. Lau (1978) has shown that the separability in inputs in the transformation function is equivalent to separability of corresponding prices in the dual profit function. The assumption of output separability simplifies the problem of multiproduct modeling by permitting outputs to be aggregated. These assumptions are particularly useful in the context of empirical work on livestock production analysis since they enable the use of various indices (for example, Törnqvist divisia index) of prices and quantities.

Nonjointness in production implies that decisions about the production of any one commodity is independent of similar decisions about other outputs. For example, in the context of livestock modeling, production of say, wool, may not be influenced by the decisions made regarding the production of broilers due to the regionality in their production. Shumway et al. (1984) discuss the causes of Nonjointness at some length. In addition to technological interdependence in the production process, the presence of allocatable fixed inputs is another important reason for the presence of jointness in production. Features of separability and Nonjointness in production are incorporated in the present study to make the model more manageable. However, assumptions on Nonjointness are statistically tested for their validity to the present model.

Stationarity

Price Expectation: In specifying the model as in Equation 3.4 current prices are assumed to prevail in perpetuity. This, indeed, is a very restrictive assumption on the part of the dynamic models. It is possible to include alternative expectation mechanisms in the empirical model. Hansen and Sargent (1980) describe a methodology to incorporate a wide class of expectations schemes in dynamic models, but at the cost of substantial complexity. Epstein and Denny (1980) incorporate output and input price expectations that are generated by first order differential equations system. However, in applying this type of expectation formation to the U.S. manufacturing data, they find that the resulting structure failed to satisfy the regularity conditions on V. Taylor (1984) documents the difficulties involved in the empirical application and interpretation of stochastic dynamic duality, particularly for the case of price expectations having a Markovian structure [1].

[1] A Markovian price expectation structure refers to any stochastic model in which price is conditional on previous prices; hence, the assumption embraces random walk, rational expectations, autoregressive and many other conditional models. The static expectation case considered in Equation 3.4 is a degenerate case of Markovian expectations.

Chambers and Lopez (1984) argue that a firm recognizing the inherent cost of acquiring information may rationally choose to form expectations statically while continuously updating the optimal policies subject to acquisition of new information. They conclude that "it seems plausible that for many small economic agents information acquisition may be costly; and it may well be *rational* to rely on static expectations."

Incorporation of *a priori* important determinants of economic behavior such as expectations into deterministics models is clearly important. The difficulty is that with generalization of each restrictive assumption (like static price expectation) a complete integration is intractable both analytically and empirically. However, one should not be satisfied with the relatively better known domain of static behavioral modeling, but should forge ahead with the more challenging dynamic models and improve the ability to approximate the real world in terms of economic models.

Constant r and δ : A constant discount rate (r) is assumed, implicitly or explicitly in many empirical studies (Schramm, 1970; Sargent, 1978; Berndt et al., 1979; Meese, 1980). The constancy of discount rate r is consistent at the aggregate level with the so-called stylized facts of economic growth (Epstein and Denny, 1980). Time-dependent discount rate r can easily be incorporated into the analysis, but that would not alter the qualitative nature of the implications of the present model. However, for simplicity, a constant r is used in the present study. The use of a constant depreciation rate, δ, is justifiable to certain extent since the study focuses on the sectoral aggregate of inventories rather than individual animals.

Technology: The value function in Equation 3.4 also assumes static technology. But, substantial technological progress has occurred in the livestock production.

Nonstationarity caused by disembodied technological change can be measured by some function, say $h(t)$, such that a time trend t can be appended to Equations 3.9-3.11 during estimation (Howard and Shumway, 1988; Vasavada and Chambers, 1986). However, the curvature properties of $h(t)$ in V are not theoretically justified. An alternative way to account for the technological progress is to use quality adjusted variables of production. For example, Gollop and Jorgenson (1980) use quality indexes for both family and hired labor. Howard and Shumway (1988) also use quality indexes for labor as well as for cows to account for embodied technical change in U.S. dairy industry. These naive approaches for modeling technical change cannot be consistent with the proposed dynamic dual model, which by definition is autonomous. As a simple alternative to the hypothesis of static technology expectations, one can assume more realistically that the firm expects a continuous and constant technical progress. Under this hypothesis, the use of time trend as a proxy for technical change can be theoretically justified by appropriate modification of the functional form chosen for V. This approach is the one adopted in the present study. In fact, any other suitable measure of technical change, say like expenditures on research and development, can be used in place of the time trend with the same theoretical justification.

To summarize this chapter, multiproduct intertemporal dual approach is adopted to model the output supply and factor demand and breeding herd adjustment decisions in the U.S. livestock sector. A normalized quadratic function is chosen to represent the present value function V. The intertemporal analogue of *Hotelling's lemma* is employed to generate the system of output supply, variable factor demand and optimal investment (inventories) functions. Sectoral level aggregate annual data are used for estimation. Among other things, static price expectation, constant depreciation rate, and constant discount rate assumptions are maintained in the present study. However, the assumption of static technological progress is relaxed in the present study by incorporating a constant and continuous technical change in a theoretically sound manner. Beef, milk, pork, chicken, eggs, turkey, sheep and lambs, and wool are the outputs considered; labor, grain feed, protein feed, hay and operating capital are the variable inputs considered; durable capital services and various livestock breeding herd inventories are the quasi-fixed inputs considered in the empirical model.

CHAPTER 4. DATA, EMPIRICAL MODEL, AND ESTIMATION

The theoretical model as discussed in Chapter 3 forms the basis for the specification of empirical model in this chapter. Firstly, a detailed description of various data series used in the study is provided. Secondly, a discussion about the empirical model and the derivation of the system of estimable equations follows. Finally, a brief description of the econometric technique employed in the estimation of the empirical model makes up the rest of this chapter.

Data

The sample period studied spans from 1950 through 1987. Livestock sector-level aggregate annual data are used in the estimation. Eight outputs, five variable inputs, and seven quasi-fixed inputs are considered in the model. Beef, milk, pork, chicken, turkey, eggs, sheep and lambs, and wool and mohair are the various outputs modeled in this study. The five variable inputs considered are, namely, operating capital, grain feed, high protein feed, hay, and hired labor. Livestock breeding herd stocks - beef cows, dairy cows, sows, chicken layers, turkey hens, and ewes and mohair goats - and fixed capital stock (durable machinery, equipments, buildings, and other structures) constitute the seven quasi-fixed inputs included in the model. The prices of all these outputs and variable inputs, user cost (also referred to as rental price) of quasi-fixed inputs, time trend, and a constant discount factor comprise the set of exogenous variables in the model. The list of model variables along with their description and source is provided in Table 4.1.

Considering the magnitude of the task of modeling the entire livestock sector, one would understandably agree that aggregation of some outputs, inputs, and their prices is inevitable. Based on the assumptions of separability in outputs and inputs, one can, for example, combine all like-outputs (their prices) into a composite aggregate output (price) index. The Törnqvist Divisia Index (Törnqvist, 1936; Diewert, 1976) is adopted in the present study to to aggregate certain outputs, inputs and their prices into suitable quantity and price indices. The Törnqvist approximation to Divisia Price Index (P_t) for a group of n commodities is given by:

$$P_t = P_{t-1} e^{\sum_{i=1}^{n} \frac{1}{2}\{(\frac{P_{it}Q_{it}}{E_{it}})+(\frac{P_{it-1}Q_{it-1}}{E_{it-1}})\} log(\frac{P_{it}}{P_{it-1}})} \tag{4.1}$$

where

P_{it} - Price of i^{th} output (input)

Q_{it} - Quantity of i^{th} output (input)

$E_t = \sum_{i=1}^{n} P_{it}Q_{it}$ - Total revenue from all outputs or total expenditures on all inputs

The implicit Quantity Index (Q_t) is given by:

$$Q_t = \frac{E_t}{P_t} \tag{4.2}$$

Note that output value shares ($\frac{P_{it}Q_{it}}{E_t}$) are used as weights in computing the indices for output quantities and prices. Similarly, input expenditure shares are used as weights for input aggregation.

Table 4.1: Description of data and model variables

Label	Description	Unit[a]	Source[b]
I. OUTPUTS			
A. Quantities:			
Y_1	Beef: Quantity of all cattles and calves produced (live weight)	Million pounds (0.01)	USDAAS
Y_2	Milk: Total milk produced. Includes fluid milk, on-farm use, and manufactured grade milk	Billion pounds	USDAAS
Y_3	Pork: Quantity of hogs produced (live weight)	Million pounds (0.01)	USDAAS
Y_4	Chicken: Commercial broilers plus other chickens produced (live weight)	Million pounds (0.1)	USDALP
Y_5	Turkey: Total production (live weight)	Million pounds (0.1)	USDALP
Y_6	Eggs: Total production	Million dozens (0.1)	USDALP
Y_7	Sheep and lambs produced (live weight)	Million pounds (0.1)	USDAAS
Y_8	Wool and Mohair: Quantity index of total wool (shorn and pulled) and mohair produced	Index	Computed

[a]Units of quantities and prices are rescaled such that their product is always in millions of dollars. This is done to facilitate consistency in aggregation. The figures in parentheses are the data rescaling factors to facilitate better efficiency in implementing the numerical algorithm in estimation. All prices are normalized by agricultural wage rate.

[b]USDAAS - *Agricultural Statistics*, USDA.

USDALP - *Livestock and Poultry Outlook and Situation*, USDA.

Computed - Derived by employing Equations 4.1-4.2

Table 4.1 (Continued)

Label	Description	Unit	Source
B. Prices:			
p_1	Beef: Average price received by farmers for beef cattle (live weight basis)	\$/cwt	USDAAS
p_2	Milk: All milk wholesale price received by farmers	\$/cwt (10.0)	USDAAS
p_3	Pork: Average price received by farmers for hogs (live weight basis)	\$/cwt	USDAAS
p_4	Chicken: Average price received by farmers for broilers(live weight weight basis)	\$/lb (10.0)	USDALP
p_5	Turkey: Average price received by farmers (live weight basis)	\$/lb (10.0)	USDALP
p_6	Eggs: Average price received by producers	\$/dozen (10.0)	USDALP
p_7	Average price received by farmers for lambs	\$/lb (10.0)	USDAAS
p_8	Wool and Mohair: Divisia index of average prices received by producers for shorn wool and mohair	Index (1950 = 1.0)	Computed

Table 4.1 (Continued)

Label	Description	Unit	Source
II. VARIABLE INPUTS			
A. Quantities:			
X_0	Labor: Total labor used in livestock production	Million hours (0.10)	USDAAS
X_1	Operating Capital: Quantity index of operating capital used in all livestock production	Index	Computed
X_2	Grain Feed: Quantity index of coarse grain feed use in livestock production	Index	Computed
X_3	Protein Feed: Quantity index of all high protein feeds (animal plus crop sources) used in livestock production	Index	Computed
X_4	Hay: All hay fed to livestock (domestic disappearance)	Million tons	USDAAS
B. Prices:			
w_0	Labor: Annual average wage rate for all hired farm workers (*numeraire*)	\$/hr (10.0)	USDAAS
w_1	Operating Capital: Price index	Index (1950=10.0)	Computed
w_2	Grain Feed: Price index	Index (1950=10.0)	Computed
w_3	Protein Feed: Price index	Index (1950=10.0)	Computed
w_4	Hay: Average price received by farmers for all hay (baled)	\$/ton	USDAAS

Table 4.1 (Continued)

Label	Description	Unit	Source
III. QUASI-FIXED INPUTS			
A. Quantities:			
Z_1	Beef cows: Total number of beef cows and heifers calved	Million heads (10.0)	USDAAS
Z_2	Dairy cows: Total number of dairy cows and heifers calved	Million heads (10.0)	USDAAS
Z_3	Sows: Total number of hogs kept on farm for breeding purposes	Million heads (10.0)	USDAAS
Z_4	Chicken layers: Total number of hens	Million heads	USDAAS
Z_5	Turkey: Total number of breeder hens in 26 major producing states	Million heads (10.0)	USDAAS
Z_6	Ewes and Angora goats: Index of total number of ewes and Angora goats clipped	Index	Computed
Z_7	Durable Capital: Quantity index of stock of durable farm machinery, equipments, buildings and structures attributable to livestock production	Index	Computed

Table 4.1 (Continued)

Label	Description	Unit	Source
B. Rental Prices:			
c_1	Beef cow: Rental price	\$/head (0.10)	Computed
c_2	Dairy cow: Rental Price	\$/head (0.10)	Computed
c_3	Sows: Rental price	\$/head (0.10)	Computed
c_4	Chicken layers: Rental price	\$/head	Computed
c_5	Turkey breeder hen: Rental price	\$/head (0.10)	Computed
c_6	Ewes and Angora goats: Rental price	Index (1950 = 1.0)	Computed
c_7	Durable capital: Rental price	Index (1950 =100.0)	Computed
IV. OTHER VARIABLES			
t	Time Trend	1951...1987	
r	Constant discount factor	0.05	
$\vec{r}$	A diagonal matrix (of required dimension) whose diagonal elements are all r		
V	Net present value of profits from all livestock production activities	Million dollars	

Outputs

Production and farm prices for beef, milk, pork, chicken, turkey, eggs, and sheep and lambs are obtained directly from various issues of *Agricultural Statistics* (USDAAS). These data are used as such in the estimation without any further manipulation. However, wool and mohair are combined as per Equations 4.1-4.2 into one composite output. The quantity of pulled wool produced is relatively small compared to shorn wool. Besides, data on pulled wool production is not available after 1981. Hence, total wool production (pulled wool plus shorn wool) is considered in constructing the price and quantity indices for wool and mohair.

The theoretical model specified in Chapter 3 maintains static price expectation. In reality, however, production decisions depend partly on firm's perception about the prices that would prevail at the time the output would be ready for marketing. Incorporating sophisticated, yet realistic, expectation formation (like rational expectations) would substantially complicate the empirical implementation of the model. Therefore, in setting up a model for practical purposes, a trade-off exists between realism and tractability. In the present study, the expected output prices (p_i, i=1,2...8 in Table 4.1) are approximated by the respective market prices lagged one period.

Variable Inputs

Incorporating all the variable inputs that are actually used in livestock production into our present model is almost impossible. However, some important ones are combined into groups to form five major variable inputs that are eventually considered in the model. Data on livestock output-specific input use are either unavailable or incomplete when available. In fact, one of the advantages of modeling production and factor demand via dual approach is that information on total input use in all the outputs considered is all that is needed to implement the method. Unfortunately though, in dual approach, the total input use cannot be allocated among various outputs that use this particular input. If such input allocation information is paramount to a researcher, one could adopt primal approach to modeling. Considering the objectives of the present study and limitations on data availability, only livestock sector-level aggregates are considered for the inputs included in the model.

Operating Capital: Total operating capital expenses on livestock production include expenditures on (i) livestock purchases (including fluid milk fed to calves and eggs used for hatching), (ii) petroleum and fuel oils, (iii) electricity, and (iv) "other" production expenses. "Other" production expenses include repairs and maintenance of capital items, machine hire, custom work, marketing, storage, transportation (including livestock marketing, milk hauling) and other *miscellaneous* expenses. The *miscellaneous* items in turn include other miscellaneous livestock purchases, livestock rental fees, health and breeding services and supplies, custom feeding and grazing, farm supplies, tools and shop equipments, net insurance and licensing fees, dairy supplies, veterinary fees, dairy assessment fees, etc.

All relevant data are available in *Economic Indicators of the Farm Sector* (USDAEI). Data on expenditures on petroleum, fuel oils, electricity, and "other" items used exclusively in livestock production are not readily available. However, information on these expenditure categories are available for the whole agricultural sector (crops and livestock) of the United States. Note that the data reported in *Economic*

Indicators of the Farm Sector on operating capital expenditures in agricultural sector are, in turn, derived based on the share of capital items used in the farm business relative to to total capital items used in the entire economy. These total expenditure figures are further broken down into expenditures attributable individually to livestock and crop production based on the relative shares of cash receipts from these two sub-sectors compared to the cash receipts from the whole of agricultural sector. For example, expenditures on electricity used in livestock production is obtained by multiplying the total expenses on electrical energy used in the agricultural sector by the ratio of cash revenues from livestock production to that from entire agricultural sector. Indices of prices paid by producers for their livestock purchases, fuels and energy, and other farm services are used along with the expenditures on these various operating capital items to construct the composite price index and implicit quantity index (using Equations 4.1-4.2) that represents the operating capital variable in the present model.

Grain Feed: This variable input category is a composite index of corn, sorghum, barley, and oats used as feed in livestock production. The prices received by producers of these feed grains are used in constructing the divisia price and quantity indices for this aggregate grain feed category. The data on feed use of these grains and respective farm prices are collected from various issues of *Agricultural Statistics* (USDAAS).

Protein Feed: High protein feed items include all oil seed meals, protein feed from animal source, wheat and rye. Total oil seed meals available for feed use includes meals from soybean, cotton seed, linseed, sunflower, and peanut, all expressed in terms of 44 per cent protein soymeal equivalence. Animal-source protein like fish meal, meat meal, and dried milk are also expressed in terms of 44 percent soymeal protein equivalence. The quantities of wheat and rye used as livestock feed are generally small; however, if and when used, they are used as protein supplements because of their higher protein content relative to other coarse feed grains. Therefore, wheat and rye are included in the high protein feed category. Prices of soymeal (44 percent protein, Decatur market), meatmeal (50 percent protein, Kansas City market), wheat and rye (average farm price) are used in aggregating these four high protein feed items into one quantity and price index. Data on individual protein feed item used and its price necessary to compute the protein feed price and quantity index are obtained from *Agricultural Statistics* (USDAAS).

Hay: Domestic disappearance of "all" hay as reported in *Agricultural Statistics* (USDAAS) is considered as the total feed use of hay in U.S. livestock production. Included in "all" hay are alfalfa, clover, timothy, wild hay, grain crops cut for hay, peanut vine, etc. For those years when the information on domestic disappearance of hay is not available, it is computed as total production of all hay plus carry-over stock less ending stock. The appropriate input price considered is the average price received by farmers for all baled hay (USDAAS).

Labor: Total hours of all labor used in all livestock and livestock product enterprises is considered as the *numeraire* input. These livestock enterprises include cattle and calves, hogs, chicken, turkey, sheep and lambs, wool, and other minor livestock (USDAAS). Family labor is not distinguished from hired labor mainly to keep the empirical model more manageable. Besides, researchers like Hertel and McKinze (1986) and Moschini (1988) argue that it is reasonable to aggregate these two labor categories under the simplifying assumption of separability in hired labor and family labor. Aggregate wage rate information for farm workers engaged in all livestock production activities is not readily available for all the years in the sample period. Therefore, annual average wage rate for all hired farm workers (crops as well as livestock) is used as the relevant input price.

Quasi-fixed Inputs

Variable inputs, by definition, are assumed to adjust completely and instantaneously for changes in external economic stimuli like changes in relative prices. However, quasi-fixed inputs are those that cannot be fully adjusted for such stimuli within the current period. Sluggish or partial response of these inputs to changing market forces can be attributed to the presence of *internal* and/or *external* adjustment costs (Penrose, 1959; Lucas, 1967). Besides, breeding herds like cows and sows cannot be adjusted (increased) instantaneously because of the inherent biological lags associated with their growth process. To adopt the adjustment cost hypothesis as an explanation for observed sluggish dynamics in livestock production, livestock breeding herds are treated as quasi-fixed inputs in the model. Specifically, beef cows, dairy cows, sows, chicken layers, turkey hens, ewes and mohair goats are considered as quasi-fixed inputs. In addition to these breeding herds, stock of durable capital used in livestock production is also included as a quasi-fixed input in the present study.

Rental Price of Quasi-fixed Inputs

In the case of variable inputs, their purchase prices can be used as appropriate input prices since all of the quantities of these variable inputs are exhausted in one period in the production process. Quasi-fixed inputs, on the other hand, render their services to production over a period of time. Therefore, their purchase prices cannot be used directly as appropriate unit input cost. Instead, one has to impute a *user cost* or *rental price* that can be attributed only to the services provided by the quasi-fixed inputs in one production period. Typically, such rental prices incorporate information like purchase price, tax rate, depreciation, salvage, etc. For instance, Branson (1979), and Moschini (1988) define imputed nominal rental price of capital good as:

$$c = PP(i + \delta + \tau) - g^e \tag{4.3}$$

where

c - rental price for the services of the capital good

PP - purchase price of the (new) capital good

i - nominal interest rate (opportunity cost of investing in new capital good)

δ - depreciation rate

τ - income tax rate

g^e - expected capital gain(loss)

Rental price calculation for a single livestock breeding animal is more complicated because of the underlying biological growth process. In particular, issues like reproduction, maintenance feeding, growth, and culling must be dealt with. Howard and Shumway (1988), for example, compute the rental price of a dairy cow as three-year amortized value of its purchase price plus discounted value of maintenance feed cost less its cull value. Tsigas and Hertel (1989) modified the formula developed by Durst and Jeremias (1984) by incorporating, among other things, taxes, to compute the rental price of a dairy cow as the annualized net income per dairy cow that would justify the investment. To incorporate all relevant information in computing a realistic rental price series for livestock would require enormous amount of technical information. In practice, therefore, researchers do make some simplifying assumptions to construct these rental prices. The following version of the formula given by Moschini (1988) is used in the present study to compute the rental prices of quasi-fixed inputs of the model:

$$c = PP(i + \delta - \beta) \tag{4.4}$$

where c, PP, i, and δ are defined as in Equation 4.3 and β is the long-term (5-10 years) moving average of actual inflation in the purchase price of the capital good to account for the capital gain (loss). This version, as simple it may be, does capture all the essential ingredients to reflect the user cost of a quasi-fixed input.

Beef cows: The beef cow herd is the underlying force in beef production dynamics. The potential breeding stock during the current year determines the following year's calf crop, which in turn, determines the steers and heifers raised for beef production. Total number of beef cows and heifers that have calved in a given year is considered as one of the seven quasi-fixed inputs. For some earlier years in the sample period, this variable defines the total number of beef cows plus heifers of age two years or older. The rental price for a beef cow is calculated using the formula in Equation 4.4. Weighted average price of a 600 pound feeder steer [1] at Kansas City market (USDAAS) is used as the purchase price (PP) for a replacement beef cow. The interest rate (i) used in the formula is the average cost of loans (interest charges plus other service fees) from production credit associations (USDAAS). A constant

[1] Weighted average price of steers is reported only on cwt. live weight basis. It is assumed that the average weight of a replacement heifer in beef cow herd is 600 lbs.

physical depreciation ($\delta = 0.15$) of the productivity of the beef cow is assumed. Finally, the capital gain (loss), β, is captured by a 10 year moving average of the actual inflation in the purchase price of feeder steers.

Dairy Cows: The treatment of total stock of dairy cows and dairy heifers that have calved as a quasi-fixed input is very similar to that of beef cows. Rental price per dairy cow is also calculated in a similar fashion except that the annual average price received by farmers for a milk cow, as reported in *Agricultural Statistics* (USDAAS), is used as the relevant purchase price (PP).

Sows: Pork production is usually modeled using quarterly data, since, the production lag associated is shorter than a year. It takes only about 5 to 6 months to finish a 40-45 pounds feeder pig to a slaughter weight of 230-250 pounds. In the present empirical model, however, annual data are used for estimation. To be consistent, pork production is also considered under this annual time period framework. The level of pork production is assumed entirely to depend upon the prevailing breeding herd size and breeding (culling) decisions. Therefore, the total number of hogs (6 months and older) kept on the farm for breeding purposes is considered as the relevant quasi-fixed input for for pork production. Rental price calculation included the seven-market average price of a 480-pounds sow [2] as the purchase price (PP), production credit association's cost of loan as the interest rate (i), a constant 0.10 as the depreciation rate (δ), and a five-year moving average of inflation in the purchase price of sow as a measure of capital gains(loss). All the relevant data for such a calculation are available in *Agricultural Statistics* (USDAAS).

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[2] Seven-market average price per sow is reported only on cwt. live weight basis. The average weight of a marketed sow is assumed to be 480 pounds. Note that the price data for the period 1950-1960 is eight-markets average (USDAAS).

Chicken Layers: The size of the hatchery flock essentially determines the production capacity in chicken industry. The subsequent stages in production follow sequentially, once the size of the hatchery flock is given. The total number of hens on farm is treated as the important quasi-fixed input in chicken meat (commercial broiler plus other) and egg production. Price series for chicken layers is not readily available. Therefore, the value of chicken (all type on farm) per head, as reported in *Agricultural Statistics* (USDAAS), is used as a proxy for the purchase price of chicken layers in rental price calculation. Average cost of loans from production credit associations as i, a constant 0.50 as δ, and annual growth in the purchase price of hen as a measure of capital gain (loss) β are used in the rental price calculation.

Turkey Hens: Total number of turkey breeder hens in 26 major turkey producing states in the U.S. defines this quasi-fixed input in turkey production. The value of turkey hen (breeder and other on farm) per head is used as the purchase price (PP). These data series are available in *Agricultural Statistics* (USDAAS) only till 1984. For subsequent years in the sample period, it is assumed that the breeder hens constituted about 2 percent of the total number of turkeys raised. Similarly, purchase price of turkey breeder hen for periods after 1984 are computed by adjusting the previous year's value by the inflation in the producer price index for livestock purchases. The other variables, (i, δ, β), used in the rental price calculation are defined similar to those in chicken layer's rental price calculation.

Ewes and Mohair Goats: Mutton and wool are joint products. The output quantity index Y_8 , considered in the model defines a composite index for wool as well as mohair. Therefore, the breeding stocks underlying the production of mutton, wool, and mohair, namely, ewes and Angora goats, are combined into one aggregate quasi-fixed input index. The total number of ewes that are at least one year old and total number of Angora goats clipped for mohair production are combined according to Equations 4.1-4.2. Note that in the case of goats clipped, it is the sum of goats and kids clipped in Spring and kids clipped in Fall. Value of sheep per head is used as the price for both ewes and goats since the relevant price series for goats is not available separately. In computing the rental price, the computed divisia price index is used as the appropriate purchase price PP. In addition, average cost of loans from production credit associations (i), a constant 0.10 depreciation rate (δ), and a five-year moving average of the inflation in purchase price index as the capital gain (loss), β are employed in the rental price formula.

Durable Capital: This quasi-fixed input (Z_7) represents the divisia quantity index of stock of durable farm machinery, equipments, buildings, and other structures that are attributable to livestock production alone. Data on total value of the various components of Z_7 are available only for the entire agricultural sector (USDAEI). Following Thirtle (1985), it is assumed here that the share of the total value of durable capital used in livestock production is proportional to the share of cash receipts from livestock sector in total cash receipts from the whole of agricultural sector. Indices of prices paid by farmers for farm machinery and other motor supplies, and building and fencing (USDAAS) are used in constructing the divisia price index and implicit quantity index (Equations 4.1-4.2). The computed divisia price index is then treated as the purchase price (PP) of durable capital for the purpose of computing the rental price of the services of this stock of durable capital. Average interest rate charged by the federal land banks on new loans (i), constant depreciation rate of 0.03 (δ), and sample period average of the annual inflation in the computed purchase price index as β are employed in Equation 4.4 to compute the rental price of this quasi-fixed input.

Other Variables

In addition to all the variables pertaining to quantities and prices of outputs, variable inputs, and quasi-fixed inputs, two other variables are also used in the empirical model. First, as described in the theoretical model in Chapter 3, a time trend t is used as a proxy for the assumed constant technological progress in livestock production. More sophisticated representation of technical progress is avoided to keep the empirical model within the realm of manageability. Note that Vasavada and Chambers (1986) and Howard and Shumway (1988) have adopted such a way to represent technical progress; however, their use of time trend in the empirical model does not follow from rigorous theoretical treatment. Following Larson (1989), time trend t is incorporated into the present model in a theoretically justifiable manner to account for the technical progress. Secondly, a constant discount factor ($r = 0.05$) is chosen to discount the future flows of revenues from livestock production to present value. This discount rate can be allowed to be time-variant which would result in a non-autonomous problem. In general, autonomous problems are easier to solve than non-autonomous ones (Kamien and Schwartz, 1981 p. 153).

Empirical Model

The empirical application of dynamic duality to model livestock sector, which is characterized by multiple outputs and multiple inputs, calls for the specification of a suitable flexible functional form for the value function V in Equation 3.4. Once a value function is chosen, inter-temporal analogue of *Hotelling's lemma* is employed to arrive at the system of equations to be estimated using the sample period data via a suitable econometric technique. Note that an "objective function" is only an approximation of the preference of the producers just as much as an econometric model is of the reality. An alternative to such an approximation is to make the objective function mathematically more complicated, although, such a step would only make empirical implementation rather difficult. Therefore, in practice, various functional forms like the quadratic, generalized Leontief, and translog are normally used to represent the optimization objective of the producers. Traditionally, assumptions on the structure of production technology meant specification of a production function involving very few parameters. With the advances in computational art, has come an increased desire for generality in representing technology. This, largely spurred by Diewert (1971), led to the use of flexible functional forms to approximate production technologies. Within the context of practicability, the functional form chosen is "flexible" when it remains as general as possible, restricts the ultimate outcome as little as possible, and above all, is easy to estimate.

A normalized quadratic functional form is used in the present study to represent the net present value function V of the model. Agricultural wage rate (w_0) is used to normalize all the prices in the model. The normalized quadratic value function is a second order Taylor series approximation to the underlying true but unknown value function. Blackorby and Diewert (1979) and Chambers (1988) have shown that the second-order differential approximation properties of a flexible functional form are preserved under the duality mapping in the context of cost function, production function, and profit (value) function.

Several researchers (Vasavada and Chambers, 1986; Moschini, 1988; Shumway et al., 1988) have demonstrated the ease with which one can represent a complex agricultural production technology via normalized quadratic profit function. For empirical application, normalized quadratic function has certain advantages over other flexible functional forms like generalized Leontief in representing a multiple output - multiple input production technology. Recall that the system of equations to determine the optimal output production, variable factor dèmand, and quasi-fixed factor investment is obtained by simply applying inter-temporal analogue of *Hotelling's lemma.* For a normalized quadratic V, the resulting output supply, variable factor demand equations are linear in parameters. Only the quasi-fixed input investment equations and the numeraire equation are nonlinear in parameters. Besides, the matrix of second derivatives of V with respect to prices (p, w, c) is constant. This provides us an easy way to check for the convexity of V in p, w, and, c as called for by the regularity conditions on the value function. One only has to verify if the the constant matrix of second derivatives is positive semi-definite or not. If one chooses to maintain convexity in estimation, imposing such a restriction is easier in this case. Note that this constant matrix, if positive semi-definite, assures local as well as global convexity. This particular advantage of normalized quadratic functional form is not available with other commonly used functional forms. Note that by construction, normalizing all prices by the numeraire price satisfies the homogeneity assumption of the value function in prices.

Keeping the practical advantages of a normalized quadratic function, the value function V is represented as follows:

$$V(p,w,c,Z,t) = a_0 + \begin{bmatrix} a_1' & a_2' & a_3' & a_4' & a_5 \end{bmatrix} \begin{bmatrix} p \\ w \\ c \\ Z \\ t \end{bmatrix}$$

$$+\frac{1}{2} \begin{bmatrix} p' & w' & c' & Z' \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ . & & & . \\ . & & & A_{34} \\ A_{41} & . & . & A_{44} \end{bmatrix} \begin{bmatrix} p \\ w \\ c \\ Z \end{bmatrix}$$

$$+ \begin{bmatrix} h_1' & h_2' & h_3' \end{bmatrix}' \begin{bmatrix} p \\ w \\ c \end{bmatrix} t \qquad (4.5)$$

where:

p is 8×1 vector of output prices

w is 4×1 vector of variable input prices

c is 7×1 vector of rental prices for the quasi-fixed inputs

Z is 7×1 vector of quasi-fixed input quantities

t is time trend

All prices (p, w, c) are normalized by agricultural wage rate w_0. The coefficients of the model variables are represented by the sub-matrices (a, A, h) whose dimensions are as follows:

$$\begin{array}{lllll} a_0 - 1\times 1 & a_1 - 8\times 1 & a_2 - 4\times 1 & a_3 - 7\times 1 & a_4 - 7\times 1 \\ a_5 - 1\times 1 & A_{11} - 8\times 8 & A_{12} - 8\times 4 & A_{13} - 8\times 7 & A_{14} - 8\times 7 \\ A_{21} - 4\times 8 & A_{22} - 4\times 4 & A_{23} - 4\times 7 & A_{24} - 4\times 7 & A_{31} - 7\times 8 \\ A_{32} - 7\times 4 & A_{33} - 7\times 7 & A_{34} - 7\times 7 & A_{41} - 7\times 8 & A_{42} - 7\times 4 \\ A_{43} - 7\times 7 & A_{44} - 7\times 7 & h_1 - 8\times 1 & h_2 - 4\times 1 & h_3 - 7\times 1 \end{array}$$

In order to apply the dynamic duality theory for empirical purpose on hand, first and second partial derivatives of the value function V with respect to its arguments p, w, c, and Z are obtained as:

$$\begin{aligned} V_p &= a_1 + A_{11}p + A_{12}w + A_{13}c + A_{14}Z + h_1 t \\ V_w &= a_2 + A_{21}p + A_{22}w + A_{23}c + A_{24}Z + h_2 t \\ V_c &= a_3 + A_{31}p + A_{32}w + A_{33}c + A_{34}Z + h_3 t \\ V_Z &= a_4 + A_{41}p + A_{42}w + A_{43}c + A_{44}Z \\ V_{Zp} &= A_{41} \\ V_{Zw} &= A_{42} \\ V_{Zc} &= A_{43} \\ V_{tp} &= h_1 \\ V_{tw} &= h_2 \\ V_{tc} &= h_3 \\ V_{ZZ} &= A_{44} \end{aligned} \tag{4.6}$$

For example, V_p is a 8×1 vector of first partials of V with respect to the 8×1 vector of normalized output prices p. V_{Zp} is the second derivative of V with respect to Z and p taken in that order. A linear aggregation rule is adopted in the present study as outlined in Chapter 3. This implies that V is affine in Z and hence, $V_{ZZ} = A_{44} = 0$. That is, the distribution of sector-level aggregate Z among individual firms within the sector does not matter for optimal decisions at the sector level. The livestock sector consists of many price taking firms and producer theory suggests that in the long run competitive equilibrium, all such firms would operate at the minimum average cost. Blackorby and Schworm (1988) adopt this type of reasoning for their linear aggregation across firms.

Applying the inter-temporal analogue of *Hotelling's lemma* the optimal output supply, variable input demand, and quasi-fixed input investment are given by the system of equations represented by Equations 3.9-3.11. However, when the argument t is included in the value function (as in Equation 4.5) to represent a constant technical progress, the Hamilton-Jacobi Equation 3.8 has to be modified as follows (Larson, 1989):

$$rV(p, w, c, Z, t) = max\{p'F(X, Z, \dot{Z}, t, \dot{t}) - w'X - c'Z + V_z\dot{Z} + V_t\dot{t}\} \tag{4.7}$$

When time trend variable t_t is used as a proxy for the "stock" of technical know-how at time period t, $\dot{t_t}$ represents the new increment (invention) to this stock. In the present case, this increment $(t_t - t_{t-1})$ is a constant (unity). The new system of equations corresponding to Equations 3.9-3.11 is as follows:

$$Y^* = \bar{r}V_p - V_{Zp}\dot{Z}^* - V_{tp} \tag{4.8}$$

$$X^* = -\bar{r}V_w + V_{Zw}\dot{Z}^* + V_{tw} \tag{4.9}$$

$$\dot{Z}^* = V_{Zc}^{-1}\{\bar{r}V_c + Z - V_{tc}\} \tag{4.10}$$

where Y^* is an 8×1 vector of optimal output supply, X^* is a 4×1 vector of optimal variable input demand, and $\dot{Z}^*$ is a 7×1 vector of optimal quasi-fixed net investment demand. The numeraire equation, namely labor demand, can be derived using Equation 4.7 as follows:

$$X_0^* = -rV^* + p'Y^* - w'X^* - c'Z^* + V_Z\dot{Z}^* + V_t\dot{t} \tag{4.11}$$

Finally, by substituting the necessary first and second partial derivatives of V from Equations 4.6 into Equations 4.8-4.11, we arrive at the following system of equations that constitutes our empirical model.

$$\begin{aligned}
Y^* &= \vec{r}(a_1 + A_{11}p + A_{12}w + A_{13}c + A_{14}Z + h_1t) - A_{14}\dot{Z}^* - h_1 & (4.12)\\
X^* &= -\vec{r}(a_2 + A_{21}p + A_{22}w + A_{23}c + A_{24}Z + h_2t) + A_{24}\dot{Z}^* + h_2 & (4.13)\\
\dot{Z}^* &= A_{43}^{-1}\{\vec{r}(a_3 + A_{31}p + A_{32}w + A_{33}c + A_{34}Z + h_3t) + Z - h_3\} & (4.14)\\
X_0^* &= -r(a_0 + a_5t) + a5 + r\{\frac{1}{2}(p'A_{11}p + w'A_{22}w + c'A_{33}c + p'A_{12}w & \\
&\quad + w'A_{21}p + p'A_{13}c + c'A_{31}p + w'A_{23}c & \\
&\quad + c'A_{32}w)\} + -\vec{r}a_4Z + a_4\dot{Z}^* & (4.15)
\end{aligned}$$

Few modifications are in order to render the above system of equations estimable. First, the quasi-fixed input net investments ($\dot{Z}_t$) are not observable readily. Therefore, they are measured discretely by ($Z_t - Z_{t-1}$) to approximate the dynamic adjustments in the stock of breeding herd and durable capital. Ideally, one should try to represent such dynamics adjustments by differential equations of higher order to reflect reality in the herd dynamics. Previous experiences by researchers in this line of modeling (Vasavada and Chambers, 1986; Howard and Shumway, 1988) do indicate that first-order differential equation is a simplistic yet adequate way of capturing the adjustments in quasi-fixed factors particularly when considering sector level model with annual data. Note that such an approximation enables us to estimate the quasi-fixed factor equations in their stock form (Z) rather than their net investment demand form ($\dot{Z}$). Second, the stock variables (Z) appearing on the right hand side of the Equations 4.12-4.15 are replaced by their respective lagged variable Z_{t-1}. These lagged values represent the relevant initial endowments that influence production and factor demand decisions in time period t. Third, the normalized output prices (p) are lagged one year to serve as (naively) expected output prices in current period production decisions. Finally, additive disturbance terms are appended to these equations to reflect room for random errors in optimization process. Incorporating all these modifications into Equations 4.12-4.15, we arrive at the following set of equations as the "complete unrestricted" empirical model which is actually estimated in the present study.

$$
\begin{aligned}
Y &= \bar{r}(a_1 + A_{11}p + A_{12}w + A_{13}c + A_{14}Z_{t-1} + h_1 t) \\
&\quad -A_{14}(Z_t - Z_{t-1}) - h_1 + e_Y && (4.16) \\
X &= -\bar{r}(a_2 + A_{21}p + A_{22}w + A_{23}c + A_{24}Z_{t-1} + h_2 t) \\
&\quad +A_{24}(Z_t - Z_{t-1}) + h_2 + e_X && (4.17) \\
Z &= A_{43}^{-1}\{\bar{r}(a_3 + A_{31}p + A_{32}w + A_{33}c + A_{34}Z_{t-1} + h_3 t) \\
&\quad +Z_{t-1} - h_3\} + e_Z && (4.18) \\
X_0 &= -r(a_0 + a_5 t) + a5 + r\{\frac{1}{2}(p'A_{11}p + w'A_{22}w + c'A_{33}c + p'A_{12}w \\
&\quad +w'A_{21}p + p'A_{13}c + c'A_{31}p + w'A_{23}c \\
&\quad +c'A_{32}w)\} + -\bar{r}a_4 Z_{t-1} + a_4(Z_t - Z_{t-1}) + e_{X_0} && (4.19)
\end{aligned}
$$

Further simplification of the above set of equations is possible by regrouping the parameters and variables. However, to preserve the exposition of how these equations are derived from a given normalized quadratic value function, they are expressed as such.

Estimation

The system of 20 equations given by Equations 4.16-4.19 is made up of eight outputs, five variable inputs (including numeraire), and seven quasi-fixed inputs. The value function V is not included in the system of equations that is finally estimated, since it will not add any additional information to the estimation process. All parameters essential to uniquely determine V are available from the system given by Equations 4.16-4.19. This is possible because the value function V is simply a linear combination of outputs and inputs, and thus, the full covariance matrix of a system in which the value function is also included would become singular (Shumway, 1983).

The functional form chosen to represent the value function must satisfy certain regularity conditions to be conformable with the dynamic duality theory (Epstein, 1981). In empirical application, appropriate statistical tests must be carried out to verify if these conditions are indeed satisfied. When situation warrants, it may be necessary to maintain these conditions (for example, homogeneity, convexity) during parameter estimation to stay within the boundaries of a sound theoretical model.

The normalized quadratic function, by construction, assures the homogeneity property of the value function. All prices (p, w, c) in Equations 4.16-4.19 are normalized by the wage rate (w_0). This implies that these equations are homogeneous of degree zero in all normalized prices while the value function itself is homogeneous of degree one in these prices. Therefore, homogeneity property is maintained throughout the estimation process of the present model. Monotonicity of the value function requires that the predicted Y, X, and Z be all non-negative for all prices. Imposing such a restriction during estimation is not resorted in practice. Once the empirical model is estimated, the parameters and predicted values of all dependent variables of the system are examined to verify if monotonicity is satisfied at each sample point. Assumption of symmetry in the cross partials of the value function, apart from being a reasonable one, enables us to reduce the number of parameters to be estimated. Note that the empirical model without the symmetry condition would have 541 parameters in the system. However, with symmetry, the total number of parameters are reduced to 370 in the full model. Symmetry implies that the parameter sub-matrices $(A_{12}, A_{21} \ldots A_{43})$ must satisfy the condition:

$$A_{ij} = A_{ji} \qquad \forall \;\; i \neq j \tag{4.20}$$

Convexity

The most important regularity condition that the value function must meet is that it be convex in all prices (p, w, c). For the current empirical model, the test for convexity boils down to verifying whether the matrix of second partial derivatives of V with respect to $p, w,$ and c is positive semi-definite or not. That is, the matrix

$$A_s = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \tag{4.21}$$

must be positive semi-definite for V to be convex in $p, w,$ and c. Note that A_s is a (19×19) matrix whose elements are constants and are independent of model variables. This means that local convexity also implies global convexity.

To test for convexity, first the system of equations (Equations 4.16-4.19) are estimated, the sub-matrix of parameter estimates A_S is formed, and then the positive semi-definiteness of A_S is verified by examining all its principal minors. For a matrix to be positive semi-definite, note that all its principal minors must be non-negative. Preliminary experiences with the model estimation indicated that convexity requirement was not satisfied. In the past, researchers have down-played the significance of such violations for lack of easier ways to incorporate such restrictions in their estimation procedure. When the empirical model is claimed to have a sound theoretical underpinning, it might as well satisfy all the underlying theoretical regularity conditions. For the estimated parameters to have valid interpretation, they must come from an empirical model that adheres to its theoretical counterpart. Hence, in the present study, convexity restriction is imposed to maintain the necessary curvature properties of the value function.

Restricting the eigen values of A_S to be non-negative is one way of imposing convexity on the empirical model. Here, the elements of A_S need not be reparameterized for estimation. However, the parametric optimization will be done subject to a highly nonlinear (in parameters) constraint that guarantees the smallest eigenvalue of A_S to be non-negative. Notice the fact that any real symmetric matrix is positive semi-definite if and only if all its eigenvalues are non-negative.

Cholesky Factorization

An alternative to the eigenvalue method of imposing convexity is the use of Cholesky factorization of the parameter matrix. There are two versions of Cholesky factorization to test and or impose convexity. First version involves decomposing A_S

into two matrices $L_{(19\times 19)}$ and $D_{(19\times 19)}$ such that $A_S = LDL'$ where L is a unique lower triangular matrix with diagonal elements all equal to one and D is a unique real diagonal matrix whose elements D_{ii} are known as *Cholesky Values.* This type of decomposition is possible only when A_S is a Hermitian[3] matrix with all its leading principal minors non-zero. In other words, this type of decomposition is possible for any real symmetric matrix but for no other real matrices. The positive semi-definiteness of A_S can then be verified simply by examining the Cholesky Values. For A_S to be positive semi-definite, all D_{ii} must be non-negative. If one decides to impose convexity, one only has to reparametrize A_S such that $A_S = LDL'$ and estimate the model subject to the restrictions that $D_{ii} \geq 0, \forall i$.

Second version of Cholesky factorization of A_S involves decomposing A_S such that $A_S = U'U$ where U is a (19×19) unique upper triangular matrix with all positive diagonal elements. Note that this type of decomposition assumes that A_S is positive definite. This version differs from the first in that the *Cholesky values* D_{ii} are all now absorbed into U. Therefore, this version imposes convexity directly rather than allowing us to test for it. Non-convex value function is inconsistent with the dynamic duality theory of producer optimization behavior. The preliminary parameter estimates indicated that the convexity assumption was not validated the empirical model. Hence, to maintain this crucial theoretical characteristic of convexity in our present model, the second version of Cholesky factorization is adopted. When A_S is reparameterized in terms of U, the total number of parameters in the estimated model remain the same. However, the equations of the model are now highly non-linear in

[3] A matrix is a Hermitian matrix if it is equal to its conjugate transpose. For a real symmetric matrix like A_S, notions of transpose and conjugate transpose are equivalent.

the new parameters making estimation more difficult.

Model estimation, hypothesis tests, validation and policy impact analysis all are contingent upon the fact that symmetry, homogeneity, and convexity assumptions are maintained throughout.

Three-stage Least Squares

The system of equations estimated involves cross-equations restrictions in terms of parameters shared by more than one equation. Quasi-fixed inputs are jointly dependent variables in the system. Considering the interdependencies among the decisions regarding production and factor demand in a multiple output - multiple input system, contemporaneous correlation among the stochastic components of the equations are highly likely. Therefore, the entire set of twenty equations is estimated as a single system using three-stage least squares procedure. A brief description of the algorithm implemented in estimation is provided below.

Consider the following stochastic version of the non-linear system (Equations 4.16-4.19) expressed in a more general form:

$$\mathbf{Y} = f(p, w, c, r, t, Z, Z_{t-1}; \theta) + \mathbf{e} \tag{4.22}$$

where Y now represents the vector of *all* twenty dependent variables of the model, θ is the set of all parameters, and $\mathbf{e}$ is the vector random disturbances.

Three-stage least squares procedure, developed by Zellner and Theil (1962) as a simple logical extension of Theil's two-stage least squares, is a systems method applied to all equations of the model simultaneously. Two of the crucial assumptions of this procedure are: (a) the random error component (e) is serially independent, that is, no autocorrelation, and (b) these random error terms are contemporaneously dependent, that is, $E(\mathbf{e_i e_j}) \neq 0$), where i and j refer to the equations in the model.

$$E(\mathbf{ee}') = \Omega \otimes I_T \tag{4.23}$$

where Ω is a 20×20 matrix of variance-covariance of the error terms $\mathbf{e}$ with a typical element $E(\mathbf{e}_i \mathbf{e}_j) = \sigma_{ij} I_T$ and T is the total number of observations in the sample.

The three stages of estimation under this procedure can be summarized as follows:

Stage I: Reduced form equations of all endogenous variables that appear on the right-hand-side of the equations in the system are estimated and the predicted values of these dependent variables are obtained. In our model, quasi-fixed inputs (Z) are such jointly dependent variables in the system. Hence, in Stage I, their predicted values ($\hat{Z}$) are generated via principal components approach using p, w, c, t, and Z_{t-1} as instruments.

Stage II: $\hat{Z}$ are substituted for Z on the right-hand-side of the structural equations and ordinary least squares procedure is applied to the transformed equations. A set of estimates for the random components ($\hat{e}$) and their variance-covariance matrix ($\hat{\Omega}$) are obtained.

Stage III: With $\hat{\Omega}$ as the appropriate weighting matrix, generalized least squares procedure is applied to the entire system of transformed structural equations to obtain the parameter estimates ($\hat{\theta}$).

Aitken type estimates for θ are obtained by minimizing error sum of squares $S(\theta)$ after the residuals are properly weighted. The weights are defined by the elements of the variance-covariance matrix Ω. More specifically, minimize:

$$S(\theta) = \frac{1}{T}\mathbf{e}'\Omega^{-1} \otimes I_T\mathbf{e} \tag{4.24}$$

When theoretical restrictions (convexity, symmetry) are imposed on the value function, the problem becomes a non-linear constrained parametric optimization as follows:

$$\begin{aligned} S(\theta) &= \frac{1}{T}\mathbf{e}'\Omega^{-1} \otimes I_T\mathbf{e} \\ \text{Subject to :} & \\ & \psi(\theta) \geq 0 \end{aligned} \tag{4.25}$$

where ψ represents the set restrictions imposed on the parameters to maintain convexity via Cholesky factorization. Since Ω is unknown, we replace it with an identity matrix and obtain preliminary estimates for θ and Ω via ordinary least squares. Ω^{-1} in Equation 4.25 is then replaced by its estimate $\hat{\Omega}^{-1}$ and new estimates for θ and Ω are obtained. This two-step procedure is repeated until $\hat{\theta}$ and $\hat{\Omega}$ stabilize. It has been shown (Judge et al., 1988) that such an estimate for $\hat{\theta}$ is asymptotically equivalent to maximum likelihood estimates at the point of convergence.

The parameter estimates ($\hat{\theta}$) are obtained by minimizing Equation 4.25 subject to the constraints set (ψ) using Davidon-Fletcher-Powell algorithm that employs numerical derivatives. GQOPT/PC,(version 5.0) - a general purpose numerical optimization package - is used in the estimation.

In summary, this chapter describes first, the development of various data series used in the empirical implementation of dynamic duality to model production in livestock sector. The data description covers eight outputs, five variable inputs, and seven quasi-fixed inputs. Secondly, the empirical model as defined by the specification of the normalized quadratic value function and subsequent derivation of the system of equations for optimal output production, variable input demand, and quasi-fixed input investment is outlined. Thirdly, incorporating theoretical restrictions such as symmetry and convexity in the estimation is described. Finally, three-stage least squares procedure is briefly described as the suitable econometric tool to estimate the empirical model.

CHAPTER 5. RESULTS AND DISCUSSION

The system of output supply, variable input demand, and quasi-fixed input stock Equations 4.16-4.19, as developed in Chapter 4, is estimated using nonlinear three-stage least squares procedure. Homogeneity, symmetry, and convexity restrictions are maintained throughout the estimation and subsequent analysis. This estimated model forms the basis for series of tests of hypotheses on the structure of dynamics in livestock herd adjustments and on nonjointness in production. The empirical estimates of the model parameters and elasticities from the accepted model are presented and appraised in this chapter. Finally, the accepted model is validated for its goodness of fit and utilized for few relevant economic stimuli impact analyses.

Tests of Hypotheses

Structure of Dynamics

Empirical model maintaining all relevant theoretical restrictions is subject to some tests of hypotheses regarding dynamics of adjustment and nonjointness in production. There are three candidates for the appropriate test statistics, namely Wald (W), Likelihood Ratio (LR), and Lagrange Multiplier (LM) with the property $W \geq LR \geq LM$. Denoting the determinants of the variance-covariance matrix of the restricted and unrestricted models as $|\hat{\Omega}_R|$ and $|\hat{\Omega}_U|$ respectively, these test statistics are defined as follows:

$$W = T \bullet (\frac{|\hat{\Omega}_R| - |\hat{\Omega}_U|}{|\hat{\Omega}_U|}) \tag{5.1}$$

$$LR = T \bullet log(\frac{|\hat{\Omega}_R|}{|\hat{\Omega}_U|}) \tag{5.2}$$

$$LM = T \bullet (\frac{|\hat{\Omega}_R| - |\hat{\Omega}_U|}{|\hat{\Omega}_R|}) \tag{5.3}$$

where T is the total number of observations. These test statistics are distributed asymptotically as χ^2 with degrees of freedom equal to the number of restrictions imposed.

One of the advantages of representing the value function of the model in normalized quadratic form is that the quasi-fixed input net investment equations assume the form of multi-variate flexible accelerator (Nadiri and Rosen, 1969) with the corresponding adjustment matrix $M = (\vec{r} + A_{34}^{-1})$. This can be seen by rewriting Equation 4.14 as:

$$\dot{Z}^* \quad = \quad M[Z - \bar{Z}(p, w, c, t)] \tag{5.4}$$

such that :

$$\bar{Z} \quad = \quad -M^{-1}A_{34}^{-1}[\vec{r}(a_3 + A_{31}p + A_{32}w + A_{33}c + h_3t) - h_3] \tag{5.5}$$

where $\bar{Z}$ is the vector of long run steady state stocks of quasi-fixed inputs. The matrix of dynamic adjustment coefficients for the present model with seven quasi-fixed inputs is given by:

$$M = (\vec{r} + A_{34}^{-1}) = \begin{bmatrix} M_{11} & M_{12} & \cdots & M_{17} \\ M_{21} & \cdot & \cdots & \cdot \\ \vdots & & & \vdots \\ M_{71} & \cdot & \cdots & M_{77} \end{bmatrix} \tag{5.6}$$

where the diagonal elements M_{ii} are referred to as "speed of adjustment" coefficients. Note that any test of hypothesis regarding the nature of dynamic adjustments among various quasi-fixed factors of the model can easily be carried out via restrictions placed on the elements of M.

One of the important objectives of the present study is to assess the nature and magnitude of the dynamic relationships between the various industries within the U.S. livestock sector. We must identify the interdependencies, if any, between various breeding herds of livestock. We must determine whether these responses in the levels of these herds are indeed sluggish, as implied by dynamic dual model. In other words, one must determine whether a simple static model as opposed to the proposed complex dynamic model is sufficient to explain production and factor use in livestock sector. To answer these questions, a series of hypotheses on the adjustment pattern is carried out and the results are summarized in Table 5.1. All three test statistics are calculated and reported.

The dynamic dual model proposed in Chapter 3 hinges upon the notion that there exists some internal costs (say, in terms of foregone output) in adjusting the current levels of breeding herds to their long run desired levels given an external economic stimuli. This is one of the explanations proposed for the observed sluggish distributed-lag pattern in livestock breeding herd adjustments. Absence of such costs

Table 5.1: Tests of hypotheses[a]

Hypotheses	Test Statistics W	LR	LM	$\chi^2_{0.01}$ Critical Value	Degrees of Freedom
Structure of Adjustments:					
1. Independent and Instantaneous adjustments. M_{ii}=-1.0 M_{ij}=0.0 $\forall i \neq j$	1.6E+08	565.25	37.00	74.92	49
2. Independent adjustments. M_{ij}=0.0 $\forall i \neq j$	4537.04	178.23	36.70	66.21	42
3. Symmetric adjustments. $M_{ij} = M_{ji}$ $\forall i \neq j$	792.70	115.06	35.35	38.93	21
4. *Realistic* adjustments. $M_{ij} = 0.0$ for some $i \neq j$	769.07	135.27	35.30	40.29	22
5. *Realistic* & symmetric adjustments.	2422.65	114.00	36.44	53.49	32
6. Reduced Model 1 M-full & symmetric $A_{14}(i,j) = 0$ & $A_{24}(i,j) = 0$ for some (i,j).	4474.02	177.71	36.70	82.29	55
7. Reduced Model 2 M-*Realistic* & asymmetric.	4175.37	175.16	36.70	83.51	56
8. Reduced Model 3 M-*Realistic* & symmetric	8343.40	200.61	36.83	95.63	66
Nonjointness in Inputs[b]:					
1. Nonjoint in short run $rV_{p_ip_j} - V_{Zp}\frac{\delta Z}{\delta p_j} = 0$	3872.79	293.99	36.99	66.21	42

[a]Symmetry and convexity of the value function V in p, w, c are maintained in all tests.

[b]Given Reduced Model 3 as the *maintained* model.

of adjustments would imply that all factors can and should be adjusted to their optimal level instantaneously. Long run dynamic analysis would then collapse to short run (all factors variable) static analysis. Therefore, the first hypothesis tested deals with verifying the validity of adjustment cost hypothesis of dynamic dual approach for modeling livestock production. Instantaneous and independent adjustments of quasi-fixed inputs implies that the adjustment matrix M must be a (negative) identity matrix that is, $M_{ii} = -1$ and $M_{ij} \neq 0 \quad \forall \quad i \neq j$. A sequential hypothesis testing procedure would be ideal to identify which of the seven quasi-fixed inputs can indeed be treated as variable inputs and which exhibit quasi-fixity. However, for present purposes, the entire set of quasi-fixed factors is tested for the appropriateness of proposed dynamic dual model. The calculated test statistics (for W and LR) exceeded the table value convincingly rejecting the null hypothesis that all factors can be treated as variable inputs. The important implication of this test is that analyzing livestock production without incorporating the interdependencies in the dynamic adjustments of various quasi-fixed inputs would be inadequate, if not inappropriate.

Second hypothesis tested deals with the postulate that the adjustment matrix M is diagonal implying univariate flexible accelerator model. Under this hypothesis, sluggish adjustments of quasi-fixed inputs are recognized, but, such adjustments are independent from each other. That is, disequilibrium in one breeding herd market would not impact other breeding herd stocks. The calculated test statistics (W and LR) resulted in rejecting the appropriateness of univariate flexible accelerator model for livestock production. The implication of this test result is that analysis of optimal supply response and input demand must account for linkages among all relevant quasi-fixed inputs of the model.

Accepting the fact that linkages between the adjustments of various livestock breeding herds do exist and must be accounted for in modeling, next logical step is to determine the nature of such linkages among the seven quasi-fixed inputs of the model. Considering the regional characteristics of livestock production in the U.S., one could postulate that some quasi-fixed inputs influence each other directly (for example, beef cows and dairy cows), while some others do not (for example, turkey hens and dairy cows). Incorporating such *a priori* information, various tests were carried out to assess the validity of selective interdependencies among the breeding herds. Although no definitive choice among alternative models emerged over the full model, one could identify, based on the calculated statistics, a tendency toward accepting a model parsimonious in parameters and yet describe the real system rather well. For instance, Reduced Models 2 and 3 had the smallest difference of calculated LR to the respective critical value. The list of hypotheses tested, by no means, exhausts all possible nested models in the system. A careful evaluation of different reduced models would perhaps facilitate the choice of a more realistic model. Thus, the Reduced Model 3 was chosen for subsequent analyses since it has the fewest parameters and yet has an adjustment matrix (M) that allows selective dependencies among various herds.

Nonjointness in Production

Technological interdependence is one of the primary causes of jointness in production (Shumway et al., 1984). Jointness in production is also confounded by the presence of inputs that can or cannot be allocated to the production of specific output (Stefanou, 1989). It is frequently difficult to allocate data on quasi-fixed inputs to the production of specific output in aggregate multiple input-multiple output studies like the present one. For example, the contribution of dairy cow breeding herd to production of milk as well as beef (say, through culling process) cannot be sorted out easily. Hence, all quasi-fixed inputs are treated as "nonallocatable" inputs in the model.

Nonjointness in outputs, less frequently found in applied literature, implies that the (short run) profit function Π be additively separable in input prices so that:

$$\frac{\delta^2 \Pi}{\delta w_i \delta w_j} = -\frac{\delta X_j^*}{\delta wi} = 0 \quad \forall \quad i \neq j \tag{5.7}$$

Thus, the j_{th} input demand does not respond to changes in i_{th} input price. Product transformation function is said to be nonjoint in inputs (more commonly found in applied research) if there exists individual production functions such that:

$$Y_i \quad = \quad f^i(X_{i1}, .., X_{in}, Z, \dot{Z}, t) \quad \forall i \tag{5.8}$$

such that :

$$X_j = \sum_{i=1}^{m} X_{ij} \quad \text{and}$$
$$F(Y_1 .., Y_m, X_1 .., X_n, Z, \dot{Z}, t) \quad = \quad 0$$

where X and Z are variable and nonallocatable quasi-fixed inputs respectively. Necessary and sufficient conditions for (static theory) nonjointness in inputs is that the static profit function Π be additively separable in output prices (Lau, 1972).

$$\Pi_{p_i p_j} = \frac{\delta^2 \Pi}{\delta p_i \delta p_j} = \frac{\delta Y_j^*}{\delta P_i} = 0 \quad \forall \quad i \neq j \tag{5.9}$$

This implies that j_{th} output supply does not respond to changes in i_{th} output price. The treatment of parametric restrictions implied by nonjointness in inputs in a dynamic dual setting merits further explanation. Stefanou (1989) provides the necessary and sufficient conditions for evaluating the presence of jointness in production for intertemporally profit maximizing firm facing adjustment costs. Recall that the unconditional supply function obtained from the value function where all quasi-fixed inputs are treated as nonallocatable is given by:

$$Y_i = rV_{p_i} - V_{Zp_i}\dot{Z}^* - V_{tp_i} \tag{5.10}$$

with the property

$$\frac{\delta Y_i}{\delta p_j} = rV_{p_ip_j} - V_{Zp_ip_j}\dot{Z}^* - V_{Zp_i}\frac{\delta \dot{Z}^*}{\delta p_j} - V_{tp_ip_j} \tag{5.11}$$

For the case of normalized quadratic value function, the third order partials in Equation 5.11 vanish yielding:

$$\frac{\delta Y_i}{\delta p_j} = rV_{p_ip_j} - V_{Zpi}\frac{\delta \dot{Z}^*}{\delta p_j} \tag{5.12}$$

Thus, in order to test and/or impose nonjointness between i^{th} and j^{th} outputs, Equation 5.12 must reduce to zero. If we adopt the static theory nonjointness restrictions (namely, the total profits are additively separable in output prices) to dynamic case, this would require $V_{p_ip_j} = 0$ for $i \neq j$. In general, the value function is not additively separable in output prices under a nonjoint technology in a dynamic dual framework (Stefanou, 1989). Note that the short run output depends on Z and $\dot{Z}$. While Z is given in the very short run and does not change for a given price change, $\dot{Z}$, however, changes as prices change. In the long run, for steady state equilibrium, $\dot{Z} = 0$ and $Z = \bar{Z}$. Long run output depends on this level of $\bar{Z}$ which in turn responds to changes in prices (See Equation 5.5). Thus, for long run nonjointness, the following must vanish:

$$\frac{\delta Y_i}{\delta p_j} = rV_{p_i p_j} + V_{Zp_i}\frac{\delta \bar{Z}}{\delta p_j} \tag{5.13}$$

In general, when interdependencies among breeding herds are captured via a non-trivial adjustment matrix, the above conditions (Equations 5.12-5.13) do not hold.

Based on *a priori* knowledge about the regionality of production in the U.S. livestock sector, nonjointness of certain outputs is hypothesized and tested. Specifically, three groups of outputs ((i) beef, milk, and mutton, (ii) chicken, turkey and eggs, and (iii) mutton and wool) are considered to exhibit jointness in production within each group but exhibit nonjointness between groups due to the technological interdependence as well as nonallocatable quasi-fixed inputs. Both short run and long run nonjointness were tested. In both cases, the null hypothesis of nonjoint production technology among these groups was rejected. Interestingly, the dynamic adjustment matrix became unstable when nonjointness was imposed. This conclusion is not surprising since various livestock enterprises in the U.S. may appear to be regional and nonjoint in nature, but the underlying interdependencies among the various breeding herd dynamics and relative factor use pattern dictates that all these enterprises are essentially interdependent. Thus, the common practice of assuming nonjointness, as done in most static models, seems a suspect.

Dynamic Adjustment Matrix

Estimates of the dynamic adjustment parameters (M_{ij}) from the "accepted" model (Reduced Model 3 in Table 5.1) are presented in Table 5.2. Notice that M in the accepted model is parsimonious in parameters and symmetric indicating selective dependencies among the breeding herd adjustments. The stability of the system requires that M be a stable matrix, that is, all its eigenvalues have negative real parts. Examination of the eigenvalues of M does meet this requirement of stability. The off-diagonal elements, M_{ij}, reflect the nature and magnitude of linkages between the various quasi-fixed factors. For example, a negative off-diagonal element like M_{12} indicates that when dairy cows herd is below its long run equilibrium value, there will be new investments in beef cow herd along with new additions to dairy cow herd. The productive resources that bolster the stock of dairy cows to its steady-state level would also benefit the beef cow herds. Similarly, a positive element like M_{54} measures the magnitude by which stock of chicken layers would lead to dis-investment in turkey breeder hens when the former is below its equilibrium value. Note that zero value for elements like M_{14} (indicated by an x in Table 5.2) are assumed during estimation of the reduced model. A zero value for M_{14} and M_{41} imply that adjustments in beef cow herd are independent of adjustments in the stock of chicken layers and vice versa.

The diagonal elements M_{ii} measure the speed with which the i^{th} quasi-fixed input will adjust toward its steady-state value from its current disequilibrium. For instance, M_{22} = -0.1076 indicates that when the current stock of dairy cows is different from its long run equilibrium value, it would take approximately ten years to complete the adjustments toward its equilibrium, provided that all other breeding stocks are at their equilibrium values. This estimate is similar to the one reported by Howard and Shumway (1988) for dairy cows (-0.09). Similar interpretations are applicable for other quasi-fixed inputs of the model. Note that the durable capital exhibits the fastest adjustment (M_{77} = -0.3097) confirming the findings of Vasavada and Ball (1988) in their analysis investment in U.S. agriculture (-0.2310). The ad-

Table 5.2: Coefficients of dynamic adjustments[a]

	Beef cows	Dairy cows	Sows	Chicken layers	Turkey breeder hens	Ewes and Angora Goats	Durable Capital
Beef cows	-0.1480	-0.0035	x	x	x	-0.0575	0.0254
Dairy cows	-0.0035	-0.1076	x	x	x	0.0091	0.0668
Sows	x	x	-0.0117	x	x	x	-0.0105
Chicken Layers	x	x	x	-0.2798	0.0214	x	0.0099
Turkey Hens	x	x	x	0.0214	-0.1259	x	-0.0120
Ewes and Angora Goats	-0.0574	0.0091	x	x	x	-0.1273	-0.0018
Durable Capital	0.0254	0.0668	-0.0105	0.0099	-0.0120	-0.0018	-0.3097

[a] An x indicates that the relevant quasi-fixed factors are hypothesized to be independent of each other in adjustments.

justment coefficients for other breeding stocks are rather small in magnitude implying sluggish adjustments. There are no comparable estimates available in the literature to assess the merit of our estimates for these quasi-fixed factors.

Parameter Estimates and Elasticities

Parameter estimates and the implied short run and long run elasticities are evaluated in this section. The entire set of parameters of the maintained model (except A_{34} which is contained in Table 5.2 as $M = (\vec{r} + A_{34}^{-1}))$ is presented in Appendix A. The reported parameters are from structural equations of the model, namely, Equations 4.16-4.19.

Recall that the parameters associated with the price variables in the model (A_S) were reparameterized via Cholesky factorization to maintain convexity. The original parameters are recaptured as some nonlinear combinations of the estimated Cholesky parameters (U_{ij}) noticing that $A_S = U'U'$. The standard errors of these parameters are computed by linearizing these nonlinear functions via Taylor series expansion, and then applying the results of variance-covariance of linear functions of random variables (Kmenta, 1986). Therefore, the reported standard errors for these parameters are only approximate.

Normalized quadratic functional form chosen for the value function V of the empirical model maintains homogeneity property by construction. Maintaining symmetry of A_S is necessary to implement Cholesky factorization of A_S to incorporate convexity of V in prices. Therefore, no tests were carried out to check for the validity of these restrictions. The necessary monotonicity conditions on the value function imply that V must be non-decreasing in output prices and non-increasing in input prices. This translates into the condition that the predicted outputs, variable inputs, and quasi-fixed inputs must all be non-negative. Model simulation with estimated parameters indicated that this indeed is the case at every point in the sample data series.

The reduced model maintaining all the theoretical restrictions fits the sample data reasonably well. The R-square and Durbin-Watson statistics (reported in Appendix Table A.1) are based on single equation regressions of actual endogenous variable on its predicted values. It should be recognized these statistics are not strictly valid for model system where lagged endogenous variables appear as explanatory variable. As such, these statistics can only be approximations and should not be relied too heavily while drawing any inferences about the model. Nevertheless, the provide a bench mark to assess the fit of the equations of the model. R-square coefficients ranged from a low 0.16 for pork production equation to a high 0.99 for dairy cows and ewes and goats stock equation. R-square for pork is too small due to the wide year-to-year fluctuations in production as well as sow herd making estimation rather difficult. Out of a total of 304 parameters in the accepted model, 228 are significant at 5 percent level of significance. Own price effects of all outputs, variable inputs demand and quasi-fixed input stocks are all significant. Parameter estimates for time trend (t) in all equations are highly significant underscoring the fact that livestock production has been experiencing some form of technological progress.

Maintaining a clear distinction between short run and long run responses to changing economic stimuli is essential for a valid interpretation of policy impacts in applied economic analysis. Such a distinction is an important feature of present dynamic dual model as opposed to the more traditional static models of livestock production. This distinction is brought about by the incorporation of equations for new net investments in quasi-fixed inputs ($\dot{Z}$) into the model. The stock of quasi-fixed inputs (Z) are given in the short run. However, the new net investments do respond to changes in relative prices in the short run. In the long run, all these stocks are at their steady-state levels and hence, $\dot{Z} = 0$. These long run steady-state levels can and would respond to changes in relative prices in the long run. These aspects are taken into account while computing various short run and long run elasticities. All elasticities are evaluated at the sample means of the model variables. Complete formulas and estimates of these elasticities for the empirical model (Equations 4.16-4.19) are presented in detail in Appendix B.

Elasticity estimates reported from livestock production models of the past have varied as widely as the method of modeling approach itself. Such differences highlight the difficulties in capturing the dynamics of herd-building in livestock production. The important elasticities estimates from the present model are compared to similar estimates found in the literature in Tables 5.3-5.8.

Examining these tables, certain general observations can be made. First, all elasticities are reasonable and lie well with in the range of estimates reported by previous researchers. Second, not many studies were as complete as the present one when modeling livestock production. Thus, comparison of various cross price effects on production, factor demand and herd adjustments are not possible. In general, own price elasticities of output supply are reasonable, more elastic in the long run than in the short run with the exception of sheep and lambs, and wool. This is not conformable with Le Chatelier's principle. However, models within the adjustment cost framework need not be coherent with this principle (Vasavada and Ball, 1988).

Table 5.3: A comparison of selected beef supply response elasticity estimates[a]

Study	Data	Features	Elasticity *w.r.t* price of	
			Beef	Other Variables
Cromarty (1959)	Annual 1929-53	• Demand & supply model of agri. & nonagri. sectors. • Livestock & feed as a sub-system.	0.037	
Reutlinger (1966)	Annual 1947-62	•Three equation model for steers, heifers, and cows.	0.162 to 0.176 (steers) -0.686 to 0.63 (heifers) -0.17 to 0.15 (all beef)	
Langemeier and Thompson (1967)	Annual 1947-63	• Demand & supply of fed and non-fed beef.	0.232 (fed) -0.552 (non-fed) 0.160 (all beef)	
Freebairn and Rausser (1973)	Annual	• Livesector model with emphasis on beef imports.	-0.13 (fed) 1.87 (non-fed) 0.32 (all beef)	
Tryfos (1974)	Annual 1951-71 Canada	• Supply and inventory formation model for beef and lamb.	-0.009	0.024 (feed)

[a]Figures in brackets are the respective long run elasticities.

Table 5.3 (Continued)

Study	Data	Features	Elasticity *w.r.t* price of	
			Beef	Other Variables
Freebairn and Rausser (1975)	Annual 1956-71	• Production, consumption inventory, trade & price for beef, pork, & poultry.	0.0 (fed) 0.61 (non-fed) 0.14 (all-beef)	
Folwell and Shapouri (1977)	Annual	• Demand, supply, inventory and imports anlysis. • Policy via imports equation.	0.04	-0.053 (corn) 0.004 (hog)
Shuib and Menkhaus (1977)	Annual 1950-74	• Includes production, price and foreign sector. • Corn feed demand & supply.	0.14 (fed) -0.966 (non-fed)	
Haack *et al.* (1978)	Quarterly 1963-75 US, Canada	• Recursive spatial equil. model for beef. • For steers & heifers supply:	-0.138 to 0.167	-0.23 (feed)
Osipna and Shumway (1979)	Annual 1956-75	• Disaggregated Supply, demand and inventory of beef. • Current *vs.* expected price effects.	2.63 to 3.16 (Choice) 0.12 to 1.34 (Good) 0.14 (all)	-0.65 to 01.03 (corn) 0.02 to 0.31 (corn) -0.25 (corn)
Brester and Marsh (1983)	Annual 1960-80	• Demand & supply of beef • Retail & wholesale. • Rational distributed lag formulation.	-0.192 (fed) -1.25 (non-fed) [-2.71]	0.424 (corn) [0.920]

Table 5.3 (Continued)

Study	Data	Features	Elasticity *w.r.t* price of	
			Beef	Other Variables
Aradhyula and Johnson (1987)	Annual 1960-82	• Demand & supply model • Incorporates Rational Expectation Hypothesis and adjustment costs concepts.	0.38 to 0.49	-0.01 to -0.05 (feed)
Grundmeier *et al.* (1989)	Quarterly 1967-86	• Supply & price determination model. • Biological restrictions via logistic function.	0.13 (fed) -0.53 (non-fed) -0.03 (all beef) [0.16]	
This Study:	Annual 1950-87	• Dynamic dual model of livestock production, factor demand, and breeding herd dynamics.	0.056 [0.096]	-0.068 (milk) [-0.116] 0.064 (grain feed) [-0.027] 0.002 (protein feed) [-0.025] -0.053 (beef cow)* [0.032]*

* With respect to the computed user cost.

Table 5.4: A comparison of selected milk supply response elasticity estimates[a]

Study	Data	Features	Elasticity w.r.t price of: Milk	Feed	Other Variables
Halvorson (1955)	Semi-annual 1931-54	• Production per cow analysis.	0.0 (summer) 0.25 (winter)		
Cochrane (1958)	Quarterly 1947-56	• Production depends on current and lagged output prices.	0.03		
Halvorson (1958)	Annual 1927-57	• Nerlovian distributed lag formulation. For 1927-57 data: For 1941-57 data:	 0.128 to 0.185 [0.398 to 0.438] 0.180 to 0.312 [0.154 to 0.88]		
Cromarty (1959)	Annual 1929-53	•Supply & Demand model of agri. & non-agri. sectors. • Livestock & feed as a sub-system.	0.212		
Ladd and Winters (1961)	Annual 1926-1956 Iowa	• Production and Stock equations.	0.06		
Kelly and Knight (1965)	Sample farm data	• Linear programing • Normative supply model. • Arc elasticities.	0.04 to 0.187		

[a]Figures in brackets are the respective long run elasticities.

Table 5.4 (Continued)

Study	Data	Features	Elasticity *w.r.t* price of		
			Milk	Feed	Other Variables
Wilson and Thomson (1967)	Annual 1947-63	• Supply & demand model.	0.003 [0.521]		
Wipf and Houck(1967)	Annual 1945-64	• An aggregate analysis.	0.04 to 0.7 [0.06 to 0.16]		
Chen *et al.* (1972)	Quarterly 1953-68 California	• Nerlovian lag formulation. • Time trend for technical progress.	0.381 [2.541]		
Prato (1973)	Annual 1950-68	• Supply & demand model.	0.006 [0.007]		
Heien (1977)	Annual 1950-69	• Production, consumption, and inventories of milk, butter & cheese.	Total Milk: 0.08	-0.28 (corn)	0.25 (*w.r.t.* dairy cow additions)
		• Cow inventory & and slaughter.	Manufac. Milk: 0.28	-0.91 (corn)	
Dahlgran (1980)	Monthly 1968-77	• Two grades of milk • 14 markets.	1.74 (Grade A) 0.897 (Grade B)		
Levins (1982)	Annual 1960-78 Mississippi	• Linear model for Grade A • Lagged profitability variables.	0.77 to 0.91 [1.55 to 1.56]		

Table 5.4 (Continued)

Study	Data	Features	Elasticity *w.r.t* price of: Milk	Feed	Other Variables
Dahlgran (1985)	Annual 1954-83	• Integrates duality and distributed lag models. • Time trend for technology.	0.12 [2.0]		
Chavas and Klemme (1986)	Annual 1960-82	• Dynamics via lag and logistic function.	0.12 [2.46]	-0.10 [-0.79]	-0.05 (slaughter cow) [-1.42]
Ball (1988)	Annual 1948-79	• Static dual approach with Translog profit function. • Quality index for technical progress.	0.642		-0.554 (hired labor) -1.998 (purchased inputs) -0.556 (durable equipments)
Howard and Shumway (1988)	Annual 1951-82	• Dynamic dual with generalized Leontief func. • Dairy sector only • Elasticities at 1982 data.	-0.075 [0.144]	-0.006 [0.002]	0.003 (labor) [-0.001] 0.078 (dairy cow) [-0.145]
Liu *et al.* (1988)	Quarterly 1970-87	Selectivity bias in multi-market setting. • Elasticities via simulation for retail supply.	Fluid Milk: 0.535 to 0.631 [0.662 to 0.723] Manufac. Milk: 0.167 to 0.170 [0.403 to 0.439]		

Table 5.4 (Continued)

Study	Data	Features	Elasticity *w.r.t* price of		
			Milk	Feed	Other Variables
Blayney and Mittelhammer (1990)	Annual 1966-85 Washington	• Static dual profit maximization framework. • Supply response due to technology *vs.* price changes.	0.893	-0.195 (Concentrate) -0.112 (hay)	-0.317 (labor) -0.089 (capital) -0.113 (cow)
This Study:	Annual 1950-87	• Dynamic dual model of livestock production, factor demand & breeding herd dynamics.	0.149 [0.168]	-0.009 (grain) [-0.11] 0.026 (protein) [0.015] -0.108 (hay) [-0.111]	0.064 (dairy cow)* [-0.013] -0.051 (beef cows)* [0.0462] -0.031 (capital)* [0.039]

*With respect to the computed user cost.

Table 5.5: A comparison of selected pork supply response elasticity estimates[a]

Study	Data	Features	Elasticity *w.r.t* price of	
			Output	Other Variables
Dean and Heady (1958)	Semi-annual 1924-37 1938-56	• Cob-web theorem to explain hog cycle.		
		• Nerlovian formulation for expected price.	0.46 to 0.65	
		• For 1924-37:	0.50 (Spring) 0.28 (Fall)	
		• For 1938-56:	0.60 (Spring) 0.30 (Fall)	
Cromarty (1959)	Annual 1929-53	• Supply & demand model of agri. & non-ag. sectors. • Livestock & feed as a sub-system.	0.13	
Harlow (1962)	Annual 1949-60	• Recursive model based on cob-web theorem.	0.56 to 0.82	
Myers *et al.* (1970)	Monthly 1949-66	• Supply & demand for pork. • Demand for beef & broilers.	-0.114	
Meilke *et al.* (1974)	Quarterly 1961-72 US, Canada	• Geometric *vs.* polynomial distributed lagged price comparisons.	0.16 to 0.24 [0.43 to 0.48]	-0.01 (feed) [-0.027 to -0.12] -0.02 (beef) [-0.04 to -0.054]

[a]Figures in brackets are the respective long run elasticities.

Table 5.5 (Continued)

Study	Data	Features	Elasticity *w.r.t* price of	
			Output	Other Variables
Tryfos (1974)	Annual 1951-71	• Supply & inventory formation model for Canada.	-0.133	-0.013 (feed)
Heien (1975)	Annual 1950-69	• Six equation model of production, slaughter, imports, and price.	0.31	0.0 (corn) [-0.095] 0.0 (soymeal) [-0.014] -0.15 (labor) [-0.156] 0.03 (Chicken-retail) [0.105]
Martin and Zwart (1975)	Quarterly 1961-72 US, Canada	• Recursive quadratic programming model of demand and supply.	0.16 [0.43]	-0.002 (feed) [-0.008]
Marsh (1977)	Annual 1953-75	• Demand & supply model of beef and pork	-0.09 (current price) 0.10 (lag price)	
MacAulay (1978)	Quarterly 1966-76 US, Canada	• Recursive spatial demand and supply equilibrium	0.0895 [0.5006]	-0.1275 (feed) -0.7136]
Chavas *et al.* (1985)	Experimental 1983	• Biological Restrictions *via* differential equation.	-0.10	0.10 (corn) 0.01 (soymeal) 1.42 (feeder pigs)

Table 5.5 (Continued)

Study	Data	Features	Elasticity *w.r.t* price of	
			Output	Other Variables
Holt and Johnson (1986)	Quarterly 1967-84	• Five equation dynamic recursive model. • Lag price is proxy for expected price.	*Total:* 0.007 [0.403]	 -0.021 (corn) [-0.463]
			Sows: -0.293 [0.122]	 0.015 (corn) [-0.476]
			Barrows & Gilts: 0.042 [0.436]	 -0.025 (corn) [-0.461]
Skold *et al.* (1988)	Quarterly 1967-86	• Supply and price determination model. • Biological restriction *via* logistic function.	0.03 [0.50]	
This Study:	Annual 1950-87	• Dynamic dual model of livestock production, factor	0.425 [0.683]	-0.194 (beef) [-0.381] -0.086 (grain feed) [0.117] 0.161 (protein feed) [0.028] 0.015 (sows)* [-0.235]

*With respect to the computed user cost of a breeding sow.

Table 5.6: A comparison of selected chicken and eggs supply response elasticity estimates[a]

Study	Data	Features	Elasticity *w.r.t* price of	
			Chicken/Eggs	Other Variables
Fisher (1958)	Annual 1915-40	•Demand & supply model of chicken, eggs & corn.	-0.18 to 0.31 [0.26]	-0.43 to 0.05 (feed) [-1.08] 0.19 (labor) [0.05]
Cromarty (1959)	Annual 1929-53	• Demand & supply model of agri. & non-agri. • Livestock & feed subsystem.	0.678 (turkey & broilers)	
Hayami (1960)	Monthly 1955-59	• Model of broilers, eggs and turkey. • Distributed lag formulation.	-0.168 [0.267] 0.364 (eggs) [0.741]	-0.153 (eggs)
Soliman (1967)	Quarterly 1955-64	• Poultry sector model. • OLS, Two-stage, LIML comparison.	Other chicken: 0.352 to 0.409 0.120 to 258 (eggs)	
Heien (1976)	Annual 1950-69	• Demand, supply & stocks of broilers, other chicken, and turkey.	0.36 (broilers) 0.06 (other chic.)	0.0 (corn) [-0.02] 0.0 (soymeal) [-0.02] -0.02 (labor) [-0.03]

[a]Figures in brackets are respective long run elasticities.

Table 5.6 (Continued)

Study	Data	Features	Elasticity *w.r.t* price of	
			Chicken/Eggs	Other Variables
Chavas (1978)	Quarterly 1965-76	• Placement, hatching and production decisions.	0.09	-0.04 (feed)
Yanagida and Conway (1979)	Annual 1960-76	• Production as function of layers, time trend etc.	0.07 (broilers) 0.38 (other chic.)	
Chavas and Johnson (1981)	Quarterly 1965-76	• Hatching, testing, production, and consumption analysis.	0.03 [0.94]	-0.089 (feed)
Chavas and Johnson (1982)	Quarterly 1965-75	• Dynamics *via* distributed lag formulation. • Lag price proxy for expected price.	0.064	-0.026 (corn)
Goodwin and Sheffrin (1982)	Monthly 1968-77	• Rational *vs.* adaptive expectation of prices.	0.99	-0.693 (feed)
Brandt *et al.* (1985)	Annual 1961-82	• Chicken and eggs model.	-0.18 (other chic.)	0.19 (corn) 0.10 (soymeal)

Table 5.6 (Continued)

Study	Data	Features	Elasticity $w.r.t$ price of	
			Chicken/Eggs	Other Variables
Aradhyula and Holt (1989)	Quarterly 1967-86	• Rational expectation hypothesis to incorporate uncertainty,	0.35	-0.058 (feed)
Jensen *et al.* (1989)	Quarterly 1967-86	• Placement, hatching, testing, and production of chicken & turkey.	0.10	
This Study:	Annual 1950-87	• Dynamic dual model of livestock production, factor demand & breeding herd dynamics.	*Chicken:*	
			0.0945	0.018 (eggs)
			[0.108]	[-0.080]
				-0.044 (grain feed)
				[-0.136]
				-0.131 (protein feed)
				[-0.056]
				0.044 (layers)
				[-0.004]
			Eggs:	
			0.063	-0.040 (chicken)
			[0.135]	[-0.057]
				0.015 (grain feed)
				[0.076]
				0.026 (soymeal)
				[-0.021]
				-0.011 (layers)
				[-0.008]

Table 5.7: A comparison of selected turkey supply response elasticity estimates[a]

Study	Data	Features	Elasticity *w.r.t* price of	
			Turkey	Other Variables
Hayami (1960)	Monthly 1955-59	• Model of broilers, eggs and turkey. • Distributed lag formulation.	0.346 [0.785]	
Soliman (1967)	Quarterly 1955-64	• Poultry sector model. • OLS, Two-stage, LIML	0.459 [0.539]	
Heien (1976)	Annual 1950-69	• Demand, supply & stocks of broilers, other chicken, and turkey.	0.56	-0.51 (broiler/feed) 0.0 (labor) [0.01]
Chavas (1978)	Quarterly 1965-76	• Placement, hatching and production decisions.	0.22	-0.15 (feed)
Yanagida and Conway (1979)	Annual 1960-76	• Production, consumption, and farm price equations.	0.28	
Chavas and Johnson (1982)	Quarterly 1965-75	• Dynamics *via* distributed lag formulation. • Testing, hatching & production.	0.064 0.21	-0.026 (corn)

[a]Figures in brackets are respective long run elasticities.

Table 5.7 (Continued)

Study	Data	Features	Elasticity *w.r.t* price of	
			Turkey	Other Variables
Brandt *et al.* (1985)	Annual 1961-82	• Production, consumption, and and stock analysis.	0.21	-0.21 (feed)
Jensen *et al.* (1989)	Quarterly 1967-86	• Placement, hatching, testing, and production of chicken & turkey.	0.14 [0.23]	
This Study:	Annual 1950-87	• Dynamic dual model of livestock production, factor demand & breeding herd dynamics.	0.117 [0.136]	0.116 (chicken) [0.115] -0.177 (grain feed) [-0.131] -0.031 (protein feed) [-0.095] -0.034 (breeder hen) [0.001]

Table 5.8: A comparison of selected lamb, sheep, and wool supply response elasticity estimates[a]

Study	Data	Features	Supply Elasticity of Lambs/Sheep	Supply Elasticity of Wool/Mohair	*w.r.t.* price of
Court (1967)	Annual 1946-61 New Zealand	• Supply model of lamb, mutton & beef.	0.05 to 0.09 (lamb)[b] [2.00]		lamb
		• Optimization of expected income flow framework.	-0.25 to -0.45 [-0.73 to -0.94]		lamb
		• Adaptive price expectation			
Witherell (1969)	Annual 1948-65	• Supply & demand model for wool.		*For the U.S.:*	
		• Nerlovian lag formumation.		0.136 to 0.145[b] [0.321 to 0.346]	wool
		• Stati price expectation.		0.048 [0.121]	lamb
		• For six major wool-producting countries.		-0.367 to -0.387 [-0.856 to -0.932]	wheat
				-0.100 [-0.221]	beef
				For Australia:	
				0.066 to 0.084[b] [0.125 to 0.276]	wool
				0.185 [0.351]	lamb
				-0.046 [-0.087 to -0.150]	wheat

[a]Figures in brackets are the corresponding long run elasticities.
[b]Own price elasticity estimates

Table 5.8 (Continued)

Study	Data	Features	Supply Elasticity of Lambs/Sheep	Supply Elasticity of Wool	*w.r.t.* price of
Freebairn[c] (1973)	Annual 1953-70 Australia	• Production, inventory and price model of beef, veal, mutton, lamb & wool	0.032[d] (lamb)	-0.700	lamb
			0.015 (lamb)	0.369[d]	wool
			-0.10 (lamb)	-0.189	beef
			−0.259[d] (mutton)	-0.102	mutton
Duane (1973)		• Model of world wool market.		0.15[d]	wool
Tryfos (1974)	Annual 1951-71 Canada	• Supply & inventory formation model.	-0.416 (sheep & lambs)		lamb
Gardner (1982)	Annual	• Wool supply model.		0.19 to 0.47[d]	wool
Whipple and Menkhaus[d] (1989)	Annual 1924-83 U.S.	• Production and inventory model of sheep industry.	0.01[d] [11.38]	0.0 [11.53]	lamb
		• Capital stock management framework.	-0.15 [4.24]	0.0[d] [4.42]	wool
		• Lamb & beef competitive.	-0.08 [-5.57]	0.0 [-5.68]	hay
			0.03 [-2.64]	0.0 [-2.78]	labor

[c]Intermediate-run (4 yr.) elasticities.

[d]Short run elasticities are for 1 yr. period while Long run elasticities are for 30 yr. period.

Table 5.8 (Continued)

Study	Data	Features	Supply Elasticity of Lambs/Sheep	Supply Elasticity of Wool	*w.r.t.* price of
This Study:	Annual 1950-87	• Dynamic dual model of livestock production, factor demand & herd dynamics. • Wool as composite index of wool and mohair.	0.825	-0.230	lamb
			[0.610]	[-0.110]	
			-0.422	1.910	Wool
			[-0.358]	[1.88]	
			0.314	-0.294	Grain feed
			[0.221]	[-0.191]	
			0.089	0.202	Protein feed
			[-0.198]	[0.192]	
			-0.517	1.195	Ewes[e]
			[-0.033]	[1.297]	

[e]With respect to user cost of the quantity index of ewes and Angora goats.

Table 5.9: Demand elasticities for variable inputs[a]

	with respect to the price of			
	Operating capital	Grain feed	Protein feed	Hay
Operating capital	-0.4704	-0.0334	0.2731	0.1000
	[-0.1900]	[0.0327]	[0.0877]	[0.0904]
Grain feed	-0.0056	-0.1446	0.0531	-0.1237
	[0.0677]	[-0.1878]	[0.0193]	[-0.0970]
Protein feed	0.5376	0.0613	-0.3968	-0.0791
	[0.6606]	[0.1341]	[-0.5136]	[-0.0519]
Hay	0.3948	-0.0699	-0.0702	-0.5634
	[0.2617]	[-0.1705]	[-0.0365]	[-0.5214]

[a]Figures in brackets are the corresponding long run elasticities.

The variable inputs demands are fairly elastic with respect to their own prices (Table 5.9). For example, the estimates are -0.14 for grain feed and -0.56 for hay. Estimates indicate that grain feed and high protein feed are substitutes, while grain feed and hay are complements. Elasticity estimates for the quasi-fixed inputs are presented in Table 5.10. Own price elasticities of quasi-fixed input stocks are elastic in the short run becoming highly elastic in the long run. The implication is that a change in relative rental prices does not evoke significant change in the utilization pattern of these factors in the short run. However, in the long run, when all quasi-fixed inputs can be adjusted freely toward their steady-state values, the utilization pattern shows very elastic response to price changes. This indeed supports the notion of quasi-fixity of breeding herd stocks in livestock production.

Table 5.10: Investment demand elasticities for quasi-fixed inputs [a]

	with respect to the user cost of						
	Beef cows	Dairy cows	Sows	Chicken layers	Turkey hens	Ewes & Angora goats	Durable Capital
Beef cows	-0.1380	0.0397	-0.0161	0.0016	0.0013	-0.0092	-0.0105
	[-1.3585]	[0.6811]	[-0.2292]	[-0.0273]	[0.0235]	[0.1008]	[-0.2120]
Dairy cows	0.0727	-0.1455	0.0161	0.0078	0.0007	-0.0015	0.1531
	[0.9477]	[-1.3755]	[0.2999]	[0.0796]	[-0.0038]	[-0.0995]	[0.8586]
Sows	-0.0773	0.0655	-0.0591	-0.0042	0.0024	0.0063	-0.0019
	[-6.5523]	[5.0648]	[-5.2388]	[-0.3272]	[0.2104]	[0.5412]	[1.4242]
Chicken layers	-0.0506	0.1429	-0.0312	-0.0775	0.0193	0.0149	-0.0926
	[-0.1467]	[0.5248]	[-0.0953]	[-0.2684]	[0.0484]	[0.0495]	[-0.3807]
Turkey hens	0.2232	-0.0192	0.0819	0.0772	-0.1264	-0.0252	-0.0774
	[1.6946]	[0.0411]	[0.5416]	[0.4533]	[-0.9726]	[-0.1687]	[-0.4503]
Ewes and Angora goats	0.0568	-0.0820	0.0289	0.0106	-0.0037	-0.0499	0.0015
	[1.1336]	[-1.0227]	[0.3471]	[0.0990]	[-0.0402]	[-0.4443]	[0.1508]
Durable capital	-0.0194	0.1337	0.0025	-0.0058	-0.0031	0.0008	-0.2568
	[-0.0243]	[0.2467]	[0.0914]	[-0.0142]	[-0.0022]	[-0.0014]	[-0.7432]

[a] Figures in brackets are the corresponding long run elasticities.

Model Validation and Simulation

The estimated model provides only an approximation to the production, factor demand, and livestock breeding herd adjustments. Such an approximation model must be evaluated to assess its validity in terms of how well the model corroborates the actual system that it approximates. Various procedures have been proposed for validating econometric models. These procedures generally examine the characteristics of individual equations, as well as examining the predictive ability of the entire system of equations. Though such good characteristics do not ensure that the entire system would indeed predict the future events accurately, many researchers do validate their model via model predictions for historical periods as a model's predictive power.

Historical dynamic simulation is one way to assess the appropriateness of the empirical model for any subsequent use for economic stimuli impact analysis. The simulation is performed using actual sample period data. Simulation is dynamic in the sense that the lagged dependent variables appearing on the right-hand-side of the equations are the lagged values obtained from the simulation rather than treating them as given explanatory variables. The performance of each equation in the system is appraised in terms of root-mean-squared-percent error (RMS%E) and Theil's forecast error decomposition. RMS%E is a measure of the absolute deviation of the predicted values from its historical values expressed in percent terms (Pindyck and Rubinfeld, 1981). The Theil's statistics, BIAS, VAR, and COV, are proportions of forecast error such that a perfect fit would mean these proportions add up to unity as BIAS=VAR=0 and COV = 1. Larger value of BIAS is an indication of systematic error as it measures deviation of averages of simulated values from actual mean values. VAR indicates the ability of the model to replicate the degree of variability of in the model variable of interest. The COV measures the unsystematic error. Finally, Theil's U, if equal to zero, implies that the value predicted by the model is exactly equal to the historical value. These simulation statistics are presented in Table 5.11. These statistics indicate that the model provides an adequate representation of the livestock production and factor demand system. The only exception is the equation for breeding sows which has an RMS%E of 38.29. and a BIAS of 0.93 indicating the problems in explaining the wide year-to-year fluctuations in pork production as well as breeding sow inventories via an annual model like the present one. In practice, many of the pork production models are based on quadratic data. All the model validation measures are reasonably good for other equations of the model.

Given that the estimated model performed fairly well under historic simulation, it can be subject to specific simulation in order to assess the dynamic properties of the model. Such properties can be readily deduced via reduced form equations if the model is linear. For nonlinear dynamic models, Fair (1980) proposes a simulation procedure to evaluate their dynamic behavior. Such a procedure is adapted here to assess the impact of three hypothetical events, namely, (i) a 10 percent permanent increase in the price of grain feed , (ii) a 10 percent permanent increase in the price of protein feed and (iii) a 10 percent permanent increase in milk price. First, a base line solution is obtained by setting all exogenous variables to their respective sample period mean values. Second, the model is simulated until the endogenous variables stabilized at steady state. Third, the exogenous price variables in question are perturbed as above, one at a time, and new simulated values of endogenous variables are generated. Finally, these new levels of endogenous variables are compared to the

Table 5.11: Historical dynamic simulation statistics for the estimated model

Depend. variable[b]	Actual mean	Predicted Mean	Correlation Coefficient	RMS%E	Theil's forecast error statistics[a] BIAS	VAR	COV	U
Y_1	356.10	355.40	0.932	5.7296	0.001	0.000	0.999	0.0288
Y_2	124.90	124.80	0.494	5.8420	0.000	0.050	0.949	0.0291
Y_3	200.30	203.00	0.125	11.7253	0.014	0.034	0.952	0.0558
Y_4	1176.90	1182.90	0.988	9.3368	0.005	0.300	0.695	0.0342
Y_5	229.00	227.50	0.966	13.4803	0.004	0.068	0.928	0.0511
Y_6	545.30	544.40	0.696	4.1495	0.002	0.018	0.980	0.0206
Y_7	112.30	114.40	0.782	19.6461	0.009	0.086	0.905	0.0958
Y_8	119.90	120.80	0.729	23.6976	0.002	0.003	0.996	0.0926
X_1	748.80	755.30	0.908	9.1213	0.009	0.014	0.976	0.0436
X_2	677.80	681.70	0.886	6.0897	0.008	0.012	0.981	0.0321
X_3	162.70	162.40	0.895	15.6485	0.000	0.004	0.996	0.0676
X_4	127.50	127.50	0.842	8.4925	0.000	0.310	0.690	0.0422
Z_1	334.50	332.50	0.867	11.7100	0.003	0.550	0.447	0.0562
Z_2	149.20	159.20	0.936	14.4925	0.251	0.010	0.739	0.0617
Z_3	82.63	112.00	0.468	38.2934	0.928	0.000	0.072	0.1561
Z_4	128.20	129.60	0.040	8.8171	0.016	0.000	0.984	0.0419
Z_5	35.20	34.28	0.426	10.1301	0.053	0.079	0.867	0.0569
Z_6	307.40	306.40	0.951	16.4413	0.000	0.508	0.491	0.0690
Z_7	200.10	206.50	0.618	16.7738	0.042	0.029	0.929	0.0762
X_0	270.10	274.50	0.991	1.1330	0.040	0.271	0.690	0.0361

[a]See text for definition of various statistics.
[b]See Table 4.1 for definition of model variables.

baseline and approximate dynamic multipliers are derived. The new solution of such exercises reached steady state usually in 20 to 40 periods. The results of these simulation exercises are summarized in terms of the implied total elasticities for selected model variables in Tables 5.12-5.14.

The responses are in general small in magnitude; nevertheless, they are in appropriate directions. For example, when the grain feed price (w_2) is increased by 10 percent, beef production initially increases nominally (short run elasticity 0.064), later declines in the long run. The implied long run beef supply elasticity with respect to feed grain price is -0.028. Own price effect on feed grain demand is in the right direction as expected. With an increase in w_2, the feed grain demand decline in the short run (-0.145) as well as in the long run (-0.199). On the other hand, protein feed demand increases in the short run (0.061) with the increase in the price of grain feed. This becomes even more elastic in the long run (0.137) implying protein feed and grain feed are substitute.

When protein feed price (w_3) is increased by 10 percent, protein feed demand decreases in the short run as well as in the long run. Grain feed, being a substitute, reponds with an increase in its demand. As for beef supply, there is a positive response in the short run, eventually turning to a negative one in the long run. Some interesting results are seen in the case of milk output and dairy cows responses for an increase in w_3. While the stock of dairy cows declines, milk output supply increases, though such increase becomes smaller in the long run. Similar seemingly inconsistent responses appear in the simulation results (Table 5.14) of a 10 percent increase in the milk price (p_2). Notice that for an increase in p_2, both beef output and beef cow herd decline throughout. However, while milk supply increases (0.150 in the short run as

Table 5.12: Simulated responses of selected model variables to a 10 percent increase in grain feed price (w_2)[a]

Periods	Outputs				Breeding Herd			Variable factors	
	Beef	Milk	Pork	Chicken	Beef cows	Dairy cows	Sows	Grain feed	Protein feed
	Y_1	Y_2	Y_3	Y_4	Z_1	Z_2	Z_3	X_2	X_3
1	0.064	-0.009	-0.086	-0.044	0.042	0.036	0.039	-0.145	0.061
2	0.054	-0.005	-0.078	-0.059	0.078	0.061	0.0.79	-0.149	0.096
3	0.046	-0.004	-0.071	-0.071	0.111	0.078	0.121	-0.153	0.119
4	0.038	-0.002	-0.066	-0.081	0.141	0.089	0.162	-0.159	0.134
5	0.032	-0.002	-0.062	-0.089	0.168	0.096	0.203	-0.164	0.143
10	0.010	-0.002	-0.049	-0.113	0.271	0.101	0.403	-0.188	0.156
20	-0.012	-0.006	-0.031	-0.129	0.381	0.071	0.762	-0.211	0.151
30	-0.021	-0.008	-0.015	-0.133	0.428	0.042	1.107	-0.218	0.148
∞	-0.028	-0.011	0.086	-0.135	0.461	-0.033	2.90	-0.199	0.137

[a]Values are approximate total elasticities with respect to grain feed price, that is, $[\frac{\%\Delta \text{ depend. var.}}{\%\Delta\ w_2}]$. These values are generated via dynamic simulation at the 1951-87 sample mean values for all exogenous variables of the model. Note that the values reported for Periods 1 and ∞ are comparable to those of short run and long run elasticities respectively as reported in tables in Appendix B.

Table 5.13: Simulated responses of selected model variables to a 10 percent increase in protein feed price (w_3)[a]

Periods	Outputs				Breeding Herd			Variable factors	
	Beef	Milk	Pork	Chicken	Beef cows	Dairy cows	Sows	Grain feed	Protein feed
	Y_1	Y_2	Y_3	Y_4	Z_1	Z_2	Z_3	X_2	X_3
1	0.002	0.026	0.163	-0.131	-0.012	-0.062	-0.014	0.0533	-0.396
2	0.001	0.023	0.151	-0.113	-0.032	-0.109	-0.032	0.048	-0.443
3	-0.000	0.022	0.143	-0.101	-0.039	-0.144	-0.051	0.045	-0.474
4	-0.002	0.021	0.137	-0.091	-0.042	-0.172	0-0.071	0.043	-0.493
5	-0.004	0.020	0.133	-0.084	-0.043	-0.195	-0.092	0.042	-0.506
10	-0.012	0.019	0.122	-0.068	-0.028	-0.268	-0.199	0.043	-0.524
20	-0.020	0.018	0.111	-0.059	0.007	-0.344	-0.395	0.042	-0.523
30	-0.023	0.016	0.102	-0.057	0.026	-0.380	-0.568	0.039	-0.521
∞	-0.026	0.015	0.046	-0.056	0.044	-0.392	-1.585	0.024	-0.516

[a]Values are approximate total elasticities with respect to protein feed price, that is, $[\frac{\%\Delta \text{ depend. var.}}{\%\Delta\ w_3}]$. These values are generated via dynamic simulation at the 1951-87 sample mean values for all exogenous variables of the model. Note that the values reported for Periods 1 and ∞ are comparable to those of short run and long run elasticities respectively as reported in the tables in Appendix B.

Table 5.14: Simulated responses of selected model variables to a 10 percent increase in milk price (p_2)[a]

Periods	Outputs				Breeding Herd			Variable factors	
	Beef	Milk	Pork	Chicken	Beef cows	Dairy cows	Sows	Grain feed	Protein feed
	Y_1	Y_2	Y_3	Y_4	Z_1	Z_2	Z_3	X_2	X_3
1	-0.068	0.150	0.227	0.049	-0.031	-0.004	0.052	0.043	-0.178
2	-0.075	0.156	0.231	0.040	-0.054	-0.010	0.104	0.041	-0.141
3	-0.082	0.160	0.234	0.031	-0.071	-0.017	0.156	0.038	-0.115
4	-0.087	0.164	0.237	0.022	-0.083	-0.025	0.206	0.034	-0.095
5	-0.092	0.167	0.240	0.014	-0.091	-0.035	0.257	0.029	-0.081
10	-0.106	0.173	0.252	-0.019	-0.100	-0.084	0.501	0.008	-0.052
20	-0.115	0.174	0.275	-0.049	-0.079	-0.167	0.945	-0.012	-0.043
30	-0.118	0.173	0.293	-0.057	-0.064	-0.216	1.337	-0.015	-0.044
∞	-0.116	0.171	0.423	-0.062	-0.054	-0.309	3.650	0.012	-0.055

[a]Values are approximate total elasticities with respect to milk price, that is, $[\frac{\%\Delta \text{ depend. var.}}{\%\Delta\ w_3}]$. These values are generated via dynamic simulation at the 1951-87 sample mean values of all exogenous variables of the model. Note that the values reported for Periods 1 and ∞ are comparable to those of short run and long run elasticities respectively as reported in Appendix B.

compared to 0.171 in the long run), dairy cow herd decline. Such an anomaly in response is rather difficult to explain under current model. Different reduced models were estimated and subjected to the same simulation exercises in order to identify inconsistencies, if any, in model variable responses. The qualitative nature of these responses did not vary from one reduced model to another.

The implied perverse responses of milk cows to price changes could arise due to several reasons like the nature of data series used, severe model restrictions like naive price expectation and linear investment rule. Another possible explanation to such an anomalous reponse is the model's inability to isolate the total effect of a price change into its short run and long run components simultaneously. In our model, the entire change in net adjustments in breeding herds ($\dot{Z}$) is explained by an "error correction mechanism" which is simply a partial adjustment of current Z_t towards an implied long run steady state equilibrium, $\bar{Z}$ (See Equations 5.4-5.5). This adjustment is not broken down into into short and long run effects of a change in exogenous variables of the model. Note that in our model the parameters necessary for computing the implied long run elasticities are essentially obtained from the short run parameters via Equation 4.18. Ideally, the model should identify these individual effects simultaneously. Following Anderson and Blundell (1982), one could respecify $\dot{Z}$ equation such that the net adjustments in the breeding herds could now be explained partly by the short run variations in the exogenous variables (ΔX_t) and the rest by a partial adjustment mechanism, $M(Z_{t-1} - \hat{Z}_{t-1})$, that would account for the long run effects simultaneously. Adopting such a specification, while enables us explain better the short run versus long run aspects of adjustments is only *ad hoc* as its structure does not follow from a sound theoretical framework. On the other hand, the structure of the net adjustment equation of our model (Equation 4.18) follows from the underlying theoretical model of inter-temporal optimization based on adjustment cost hypothesis. Thus, one must investigate ways of reconciling these alternative approaches in order to explain the short run and the long run aspects of net adjustments in livestock breeding herds for a given exogenous shocks.

CHAPTER 6. SUMMARY AND CONCLUSIONS

The task of adequately modeling livestock production is confounded by, among other things, the characteristic biological lags underlying the growth process of livestock that dictates the observed dynamics of herd adjustments. The presence of some costs of adjustments (internal and/or external) can also be attributed to the observed pattern of sluggish responses in these stocks for changes in external economic stimuli like relative price changes.

There are several approaches to modeling livestock production with its distinguishing dynamic features. In practice, however, many researchers have adopted simple static reduced-form type model for production analysis. Such simplistic approach only utilizes *ad hoc* restrictions on these relationships to explain the underlying structure of production. These models seldom adhere to the theoretical restrictions implied by relevant economic theory of producer behavior. In addition, not many studies in the past have taken into account the importance of interlinkages of output supply, factor demand, and investment demand in a multiple output framework that characterizes the livestock sector.

Realizing the need for an empirical model whose structure is explicitly derived from the economic theory of firms, the present study was carried out with two main objectives. The first objective is to develop a theoretical structural model of livestock production that would capture the dynamics of adjustments among various interdependent breeding stocks and to identify and assess the necessary theoretical restrictions. The second objective is to demonstrate the empirical applicability of this model without sacrificing any of its theoretical features. Utilizing only the derived structural relationships and not adhering to required theoretical parametric restrictions (like homogeneity, symmetry, and convexity), as often done in applied research, is rather hard to justify.

A theoretical model was developed in Chapter 3 based on the adjustment cost hypothesis and intertemporal optimization of producer behavior. It was postulated that the presence of some internal cost of adjustments was the key behind the observed sluggish adjustments in the quasi-fixed inputs of livestock production. Besides, the biological lags associated with the growth of animals constrain the responses of these stocks to outside economic forces. The distinction between short run and long run behavioral responses of producers can be modeled in a theoretically consistent fashion under the adjustment cost hypothesis. Therefore, the concept of internal adjustment cost hypothesis was integrated into a multiple output-multiple input model of livestock sector. Issues in implementing this model like theoretical restrictions (homogeneity, symmetry, and convexity), aggregation, expectation formation, separability, and nonjointness in production were identified and appraised.

The data series used, empirical counterpart of the proposed theoretical model, and the econometric estimation procedure used are provided in Chapter 4. Considering the relative merits of representing a multiple output-multiple input production technology via dynamic dual framework, a normalized quadratic value function was adopted for the present study. Applying the results of intertemporal analogue of *Hotelling's lemma* (McLaren and Cooper, 1980; Epstein, 1981), the system of equations for optimal output supply, variable input demand, and quasi-fixed input investments was derived.

Beef, milk, pork, chicken, turkey, eggs, lambs and sheep, and wool (including mohair) were the eight livestock outputs considered in the model. The five variable inputs of the empirical model were namely, labor (*numeraire*), operating capital, grain feed, protein feed, and hay. Various breeding herd stocks like beef cow, dairy cow, sows, chicken layers, turkey breeder hens, and ewes (including Angora goats) were considered as the relevant nonallocatable quasi-fixed inputs of the model. In addition, the stock of durable capital attributable to livestock production was also included in the model. Annual sector-level aggregate data (1950–1987) were used for the estimation. Nonlinear three-stage least squares procedure was used in the estimation process to account for the contemporaneous correlation among the random error components of the equations of the model. To ensure that the empirical model truly adheres to its theoretical counterpart, all relevant parametric restrictions (homogeneity, symmetry, and convexity) were incorporated into the estimation process throughout. Specifically, Cholesky factorization approach was used to maintain convexity (in prices) of the value function.

The results of the present study are discussed in Chapter 5. The estimated model was subject to a series of tests of hypotheses on the structure of dynamic adjustments and nonjointness in production. Results indicated that a static model where all factors of production are variable, would not be adequate to explain the livestock production relationships. In addition, the results also indicated that all of the quasi-fixed inputs of the model exhibited some interdependencies underscoring the validity of the present modeling approach. The estimates of the adjustment matrix quantified the nature and magnitude of these interlinkages among the livestock breeding herds.

The test on nonjointness in production is more complicated under the dynamic dual framework unlike under static modeling framework because of the presence of (allocatable/nonallocatable) quasi-fixed inputs. Simple additive separability of the value function in output prices ($V_{p_i p_j} = 0$) does not imply nonjointness among i^{th} and j^{th} outputs under dynamic setting. Following Stefanou (1989), nonjointness in selected groups of outputs was tested and categorically rejected. The results indicated that livestock production in the U.S., though regional in nature, indeed is interdependent due to the underlying linkages among the dynamics of herd adjustments.

The estimated model performed well in explaining the sample period with the exception of pork production. The parameter estimates for the time trend variable were all highly significant in all equations implying that some technical progress did occur in livestock production during the sample period. Estimates of various short run and long run elasticities are calculated and appraised in Chapter 5. The important distinction between short run and long run responses was recognized in computing various elasticities. In general, elasticity estimates were comparable to similar estimates from other studies from the past.

The main conclusion that could be drawn from the present exercise is that some economic theory of producer behavior can be realistically and fully adopted while modeling livestock production relationships. One need not be content with the hitherto common practice of modeling under either static framework or "dynamic" framework where the so called dynamics is governed by an arbitrary lag structure imposed from outside without any heed to relevant theoretical restrictions of economic theory.

The present assiduous effort, while in general, is a step in the right direction of reconciling empirical modeling with theoretical justification, does have room for improvements. First, the adopted theoretical model hinges upon the crucial assumption of stationarity of expectation regarding prices, and discount factor. A logical improvement of the present work is to explore ways of incorporating better representation of expectation formation (like rational expectations hypothesis) without sacrificing the empirical applicability of the model. Second, the dynamics of livestock breeding herd adjustment is represented by a simple linear first order differential equation. In our empirical model this was approximated by the year-to-year variation in the level of stocks. In reality, however, livestock production is characterized by dynamics that are more complicated by the biological lags associated with the growth of the animals. Future modeling efforts must reap the rich information embedded in these biological lags to specify a more realistic dynamics of the herd adjustment process. Notwithstanding these weaknesses, the present model demonstrates the merits of successfully incorporating theory into practice to model livestock production.

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APPENDIX A. Parameter Estimates of the Empirical Model

In this section, all relevant parameter estimates of the empirical model (Equations 4.16-4.19) are presented.

Table A.1: Estimates of a & h parameter vectors, R^2, and DW of the model*[a]

Depend.	a		h		R^2	DW
variable	Estimate	Std. Error	Estimate	Std. Error		
Outputs:						
Y_1	-197945	1.19381	105.42	1.13955	0.90	0.48
Y_2	-28232	1.19381	15.58	0.55481	0.30	0.45
Y_3	-41488	1.19381	22.86	1.15513	0.16	1.01
Y_4	-1577732	1.19381	821.80	1.19293	0.98	1.60
Y_5	-357680	1.19381	186.28	1.17756	0.94	1.14
Y_6	-121213	1.19381	67.48	1.15399	0.49	0.80
Y_7	101483	1.19381	-50.93	1.14618	0.73	0.86
Y_8	-329873	1.19381	165.99	1.18104	0.60	1.70
Variable inputs:						
X_1	558866	1.19381	-294.21	1.18828	0.86	0.96
X_2	289553	1.19381	-156.52	1.18591	0.82	1.54
X_3	192858	1.19381	-100.14	1.13226	0.84	1.08
X_4	46035	1.19381	-25.04	0.72593	0.78	1.12
Quasi-fixed inputs:						
Z_1	-163784	1.19381	80.89	1.18678	0.97	0.71
Z_2	-95562	1.19381	48.72	1.18547	0.99	1.34
Z_3	-55495	1.19381	29.34	1.19486	0.59	1.73
Z_4	13620	1.19381	-7.64	1.16057	0.40	1.73
Z_5	19499	1.19381	-10.15	1.10567	0.34	1.75
Z_6	-12231	1.19381	3.00	1.18974	0.99	0.36
Z_7	-107157	1.19381	51.90	1.17706	0.83	1.30
Numeraire - Labor:						
X_0	-351348	1.19381	179.04	1.16727	0.97	1.83

[a]*See Table 4.1 for variable definition. a is associated with intercept. h is associated with the time trend t of the model. R^2 and DW (Durbin-Watson) statistics are from single equation regression of dependent variable on its corresponding predicted values (See Moschini, 1988).

Table A.2: Parameter estimates of A_{11} matrix of the model[a]

Depend. variable	Right-hand-side variables p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
Y_1	336.3309							
	38.5777							
Y_2	-190.3480	107.9772						
	14.8407	11.5257						
Y_3	-511.6690	280.4000	1210.4048					
	33.9333	18.4709	64.2527					
Y_4	277.6576	-149.3671	-967.9514	16002.8133				
	26.7700	15.0749	47.9313	301.7437				
Y_5	47.7315	-8.9792	-939.1116	3590.2908	3045.8254			
	17.8428	11.6364	55.3570	159.5534	131.3075			
Y_6	193.0773	-93.4288	-961.3627	-3852.8115	-253.9040	4350.8761		
	24.2136	14.1267	52.1911	162.8816	96.0496	156.8342		
Y_7	-224.0996	148.6858	-368.8078	-919.1585	3162.3076	-973.0700	8305.7779	
	25.4246	14.4826	61.0180	158.3935	117.2854	127.8799	217.0229	
Y_8	-26.6161	1.9546	244.7581	4245.0898	-968.9027	-376.7718	-5017.7042	66815.1929
	21.2763	12.5116	51.9426	162.4991	102.1727	111.5143	134.7682	617.1584

[a] A_{11} is symmetric. See Table 4.1 for variable definition. Respective Standard errors are reported immediately below the parameter esimates.

Table A.3: Parameter estimates of A_{12} matrix of the model[a]

Depend. variable	Right-hand-side variables w_1	w_2	w_3	w_4
Y_1	-95.1939	42.9861	-118.4707	246.2597
	13.6049	9.0943	14.0471	17.8845
Y_2	65.8358	-19.4992	54.1380	-139.9232
	6.2819	5.3343	8.3775	6.4132
Y_3	113.6408	-202.2024	549.4080	-385.4323
	45.4268	23.2490	34.9634	24.3305
Y_4	2313.8636	-3743.8454	-1971.6066	-1398.3175
	158.3651	148.3790	134.3489	101.9324
Y_5	-105.2742	-876.9525	-472.5381	-440.1080
	95.1443	73.1218	74.1874	47.6107
Y_6	-407.1084	962.5170	-147.7368	806.1324
	100.0791	92.4274	83.5074	45.7385
Y_7	710.5082	942.1777	-346.3930	-913.2854
	123.8431	118.6717	111.7862	65.7116
Y_8	-2319.2416	-790.2874	544.8001	1730.4558
	288.8521	258.4888	290.4731	226.4395

[a]See Table 4.1 for variable definition. Respective standard errors are reported immediately below the parameter estimates.

Table A.4: Parameter estimates of A_{13} matrix of the model[a]

Depend. variable	Right-hand-side variables						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Y_1	-208.2984	-145.5427	652.1399	336.6791	464.3075	-905.8420	-134.0043
	23.3372	18.9362	42.9277	28.3838	34.4526	56.3190	17.2733
Y_2	136.2841	79.1674	-380.3345	-208.1430	-283.6410	511.0759	83.4317
	14.0812	10.2804	23.0810	14.7348	18.5879	30.3025	10.7189
Y_3	-1333.4210	519.2205	-1070.2168	174.4734	380.0392	2225.2793	-9.4767
	83.2150	34.9369	66.5153	58.3382	68.5826	80.9677	35.3016
Y_4	9029.1952	-3130.5798	3215.6752	2244.3984	2356.5199	-9316.1033	3391.6849
	204.7476	148.8503	170.5543	159.5957	169.5442	198.1280	143.6214
Y_5	4011.6562	-491.4499	284.7843	-1128.3384	-981.1343	1271.8945	122.4848
	154.6131	73.0734	101.1445	84.0556	102.6090	220.0417	80.7769
Y_6	656.6386	308.7106	144.4203	-1943.7330	-3629.9707	-5279.1071	-489.3154
	161.1996	86.5557	113.2041	97.0685	115.8082	240.8887	92.1472
Y_7	-2655.3121	1554.6316	-4050.2114	-1523.7111	-911.3858	13463.3403	-503.2520
	189.3888	114.7460	140.1987	125.0901	139.6569	252.8895	120.4299
Y_8	3742.1505	-1084.1726	-2544.5037	-5336.5324	10812.2148	-21986.7685	4707.7151
	339.8554	303.9788	321.5603	316.2830	324.7975	388.1206	305.3303

[a]See Table 4.1 for variable definition. Respective standard errors are reported immediately below the parameter esimates.

Table A.5: Parameter estimates of A_{14} matrix of the model[a]

Depend. variable	Right-hand-side variables						
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7
Y_1	-0.1842	-0.6226	x	x	x	0.8103	-0.0475
	1.1636	1.2093	x	x	x	1.1836	1.1828
Y_2	-0.1367	0.2428	x	x	x	-0.0471	-0.0118
	0.9707	1.1678	x	x	x	1.1307	1.0913
Y_3	x	x	1.8635	x	x	x	-0.2704
	x	x	1.1985	x	x	x	1.1862
Y_4	x	x	x	1.2287	-10.8640	x	2.2279
	x	x	x	1.1938	1.1940	x	1.1942
Y_5	x	x	x	1.3415	-0.5905	x	-0.4981
	x	x	x	1.1935	1.1939	x	1.1899
Y_6	x	x	x	0.7462	2.7895	x	-0.6580
	x	x	x	1.1977	1.1973	x	1.1841
Y_7	0.0261	1.9129	x	x	x	0.6185	0.0630
	1.1795	1.1968	x	x	x	1.1914	1.1939
Y_8	x	x	x	x	x	-0.2774	-0.2302
	x	x	x	x	x	1.1920	1.1938

[a]See Table 4.1 for variable definition. Respective standard errors are reported immediately below the parameter esimates. An x indicates that that particular parameter is restricted to be zero in the "accepted" model.

Table A.6: Parameter estimates of A_{22} matrix of the model[a]

Depend. variable	Right-hand-side variables w_1	w_2	w_3	w_4
X_1	5024.8092			
	159.4273			
X_2	-48.4493	3696.7646		
	100.5552	139.3717		
X_3	-2479.9384	-654.1200	1940.3501	
	87.6529	82.9539	94.7733	
X_4	-839.5182	506.5331	108.1640	651.0692
	43.1164	42.4185	29.6613	41.5282

[a] A_{22} is symmetric. See Table 4.1 for variable definition. Respective standard errors are reported immediately below the parameter estimates.

Table A.7: Parameter estimates of A_{23} matrix of the model[a]

Depend. variable	Right-hand-side variables						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
X_1	-4026.3055	217.6647	-2746.1558	128.9994	151.9558	-537.0019	2023.2286
	177.1788	88.1064	118.0265	108.2736	124.2401	234.0723	87.3320
X_2	-3143.4225	-229.0889	-1695.2101	-333.8994	-863.8542	4224.8169	996.6050
	154.0850	80.0450	103.6877	96.8256	110.3713	230.7481	87.7091
X_3	27.8551	811.4200	729.1460	12.6208	262.6652	1937.6105	-1581.9332
	150.9395	60.1235	89.5537	92.7415	111.3937	241.1950	72.9269
X_4	495.4084	-262.5651	747.1695	-151.4126	273.2471	-1644.3360	-329.9367
	71.5682	27.6236	56.3318	64.1757	66.7732	118.2873	32.4113

[a]See Table 4.1 for variable definition. Respective standard errors are reported immediately below the parameter esimates.

Table A.8: Parameter estimates of A_{24} matrix of the model[a]

Depend. variable	Right-hand-side variables						
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7
X_1	-0.5590	-3.5093	-2.0498	0.1385	1.0662	-1.3252	1.6580
	1.1939	1.1938	1.1939	1.1942	1.1938	1.1931	1.1937
X_2	0.9483	-0.0549	-1.5331	-0.7231	5.1543	-0.4214	0.4608
	1.1964	1.1957	1.1940	1.1938	1.1939	1.1946	1.1932
X_3	0.0553	0.3120	0.1828	-1.4287	0.1414	-0.0229	0.7870
	1.1777	1.2124	1.2021	1.1926	1.1931	1.1725	1.1750
X_4	0.3285	0.3284	x	x	x	-0.1004	-0.0223
	0.8596	1.1091	x	x	x	1.1397	1.0620

[a]See Table 4.1 for variable definition. Respective standard errors are reported immediately below the parameter esimates. x indicates the parametr restricted to be zero in the *accepted* model.

Table A.9: Parameter estimates of A_{33} matrix of the model[a]

Depend. variable	Right-hand-side variables						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Z_1	22402.4704						
	355.5415						
Z_2	-6217.7246	6042.2659					
	185.6099	183.1566					
Z_3	7805.7065	-3672.7328	13556.9544				
	203.1437	131.3508	277.7986				
Z_4	1335.7124	-2696.5238	1878.0962	10886.9139			
	203.0199	135.3917	171.9929	248.4599			
Z_5	-3346.9588	43.2464	-2494.4238	-4432.3023	18349.3048		
	207.3357	144.1055	182.3260	192.8696	323.3043		
Z_6	-15235.9777	9095.5894	-11260.9635	-7393.0109	5882.6048	122305.5273	
	313.1466	386.7401	405.2516	400.7757	423.0903	834.7415	
Z_7	1248.2487	-2801.9349	-568.8717	1201.1399	342.8551	-666.5421	3799.0262
	160.7253	111.0981	141.7057	139.7694	171.3538	407.9355	138.4618

[a] A_{33} is symmetric. See Table 4.1 for variable definition. Respective standard errors are reported immediately below the parameter estimates.

APPENDIX B. Derivation of short run and long run elasticities

In this section, formulas[1] used to calculate the short run and long run elasticities are derived for the normalized quadratic value function maintaining homogeneity, symmetry, and convexity. calculated elasticities as reported in Tables B.1-B.9 are evaluated at the mean values of the model variables.

Short run (ϵ)

In the short run, Z_{t-1} appearing on the right-hand-side of the model equation 4.16-4.19 are treated as initial endowments for the current period production decisions. However, current period net investment ($\dot{Z}_t$) do change as its arguments (p, w, c, r, t, Z_{t-1}) change. Keeping these points in mind, the short run elasticities are calculated as follows:

Outputs:

$$\epsilon_{Yp} = [\vec{r} * A_{11} - A_{14} * (A_{34} * \vec{r} * A_{31})] \# (p * Y'^{-1})$$

$$\epsilon_{Yw} = [\vec{r} * A_{12} - A_{14} * (A_{34} * \vec{r} * A_{32})] \# (w * Y'^{-1})$$

[1]See Chapter 4 for definitions of model variables and the dimensions of matrices that are used in the formulas. The formulas are in matrix notation and represent the elasticity of first subscript variable with respect to second subscript price.

$$\epsilon_{Yc} = [\vec{r} * A_{13} - A_{14} * (A_{34} * \vec{r} * A_{33})] \# (c * Y'^{-1})$$

$$\begin{aligned}\epsilon_{Yw_0} = {} & [((-1/w_0) * (\vec{r} * (A_{11} * p + A_{12} * w + A_{13} * c))) - A_{14} * ((-1/w_0) \\ & *(A_{34} * \vec{r} * (A_{31} * p + A_{32} * w + A_{33} * c)))] \# (w_0 * Y'^{-1})\end{aligned}$$

Variable inputs:

$$\begin{aligned}\epsilon_{Xp} &= [-\vec{r} * A_{21} + A_{24} * (A_{34} * \vec{r} * A_{31})] \# (p * X'^{-1}) \\ \epsilon_{Xw} &= [-\vec{r} * A_{22} + A_{24} * (A_{34} * \vec{r} * A_{32})] \# (w * X'^{-1}) \\ \epsilon_{Xc} &= [-\vec{r} * A_{23} + A_{24} * (A_{34} * \vec{r} * A_{33})] \# (c * X'^{-1}) \\ \epsilon_{Xw_0} &= [(1/w_0) * \vec{r} * (A_{21} * p + A_{22} * w + A_{23} * c)) + A_{24} * ((-1/w_0) \\ & \quad *(A_{34} * \vec{r} * (A_{31} * p + A_{32} * w + A_{33} * c))] \# (w_0 * X'^{-1})\end{aligned}$$

Quasi-fixed inputs:

$$\begin{aligned}\epsilon_{Zp} &= [A_{34} * \vec{r} * A_{31}] \# (p * Z'^{-1}) \\ \epsilon_{Zw} &= [A_{34} * \vec{r} * A_{32}] \# (w * Z'^{-1}) \\ \epsilon_{Zc} &= [A_{34} * \vec{r} * A_{33}] \# (c * Z'^{-1}) \\ \epsilon_{Zw_0} &= [(-1/w_0) * (A_{34} * \vec{r} * (A_{31} * p + A_{32} * w + A_{33} * c))] \\ & \quad \# (w_0 * Z'^{-1})\end{aligned}$$

Numeraire:

$$\epsilon_{X_0 p} = [\vec{r} * (A_{11} * p + A_{12} * w + A_{13} * c) + (A_{13} * \vec{r} * A_{34}) * a_4] \# (p/X_0)$$

$$\epsilon_{X_0 w} = [\vec{r} * (A_{21} * p + A_{22} * w + A_{23} * c) + (A_{23} * \vec{r} * A_{34}) * a_4] \# (w/X_0)$$

$$\epsilon_{X_0 c} = [\vec{r} * (A_{31} * p + A_{32} * w + A_{33} * c) + (A_{33} * \vec{r} * A_{34}) * a_4] \# (c/X_0)$$

$$\begin{aligned} \epsilon_{X_0 w_0} = {} & [-(2/w_0) * ((\vec{r} * ((1/2) * (p' * A_{11} * p + w' * A_{22} * w + c' * A_{33} * c)) \\ & + p' * A_{12} * w + p' * A_{13} * c + c' * A_{32} * w) + a_4 * ((-1/w_0) * A_{34} * \vec{r} \\ & * (A_{31} * p + A_{32} * w + A_{33} * c))] * (w_0/X_0) \end{aligned}$$

Long run (ε)

Long run steady-state equilibrium $\bar{Z}$ (Equation 5.5) is obtained by recognizing the fact for long run equilibrium, $\dot{Z} = 0$ since $Z_{t-1} = Z_t = \bar{Z}$. Thus, Equation 5.5 is substituted for Z_t and Z_{t-1} on the right-hand-side of the Equations 4.16-4.17 to get corresponding long run output supply and variable factor demand equations. These implied long run equations (along with Equation 5.5) are used to derive the following formulas for long run elasticities:

Output:

$$\varepsilon_{Yp} = [\vec{r} * A_{11} + \vec{r} * A_{14} * (-(M^{-1} * A_{34} * \vec{r} * A_{31}))] \# (p * Y'^{-1})$$

$$\varepsilon_{Yw} = [\vec{r} * A_{12} + \vec{r} * A_{14} * (-(M^{-1} * A_{34} * \vec{r} * A_{32}))] \# (w * Y'^{-1})$$

$$\varepsilon_{Yc} = [\vec{r} * A_{13} + \vec{r} * A_{14} * (-(M^{-1} * A_{34} * \vec{r} * A_{33}))] \# (c * Y'^{-1})$$

$$\begin{aligned} \varepsilon_{Yw_0} = {} & [((-1/w_0) * (\vec{r} * (A_{11} * p + A_{12} * w + A_{13} * c))) + A_{14} * \vec{r} \\ & * M^{-1} * ((1/w_0) * (A_{34} * \vec{r} * (A_{31} * p + A_{32} * w + A_{33} * c)))] \# (w_0 * Y'^{-1}) \end{aligned}$$

Variable inputs:

$$
\begin{aligned}
\varepsilon_{Xp} &= [-\vec{r} * A_{21} - \vec{r} * A_{24} * (-(M^{-1} * A_{34} * \vec{r} * A_{31}))]\#(p * X'^{-1}) \\
\varepsilon_{Xw} &= [-\vec{r} * A_{22} - \vec{r} * A_{24} * (-(M^{-1} * A_{34} * \vec{r} * A_{32}))]\#(w * X'^{-1}) \\
\varepsilon_{Xc} &= [-\vec{r} * A_{23} - \vec{r} * A_{24} * (-(M^{-1} * A_{34} * \vec{r} * A_{33}))]\#(c * X'^{-1}) \\
\varepsilon_{Xw_0} &= [(1/w_0) * \vec{r} * (A_{21} * p + A_{22} * w + A_{23} * c)) - \vec{r} * A_{24} \\
&\quad *((1/w_0) * M^{-1} * (A_{34} * \vec{r} * (A_{31} * p + A_{32} * w + A_{33} * c))]\#(w_0 * X'^{-1})
\end{aligned}
$$

Quasi-fixed inputs:

$$
\begin{aligned}
\varepsilon_{Zp} &= -[M^{-1} * A_{34} * \vec{r} * A_{31}]\#(p * Z'^{-1}) \\
\varepsilon_{Zw} &= -[M^{-1} * A_{34} * \vec{r} * A_{32}]\#(w * Z'^{-1}) \\
\varepsilon_{Zc} &= -[M^{-1} * A_{34} * \vec{r} * A_{33}]\#(c * Z'^{-1}) \\
\varepsilon_{Zw_0} &= [(1/w_0) * M^{-1} * (A_{34} * \vec{r} * (A_{31} * p + A_{32} * w + A_{33} * c))]\#(w_0 * Z'^{-1})
\end{aligned}
$$

Numeraire:

$$
\begin{aligned}
\varepsilon_{X_0p} &= [\vec{r} * ((A_{11} * p + A_{12} * w + A_{13} * c) + (A_{13} * \vec{r} * A_{34} * M^{-1} * a_4))]\#(p'/X_0) \\
\varepsilon_{X_0w} &= [\vec{r} * ((A_{21} * p + A_{22} * w + A_{23} * c) + (A_{23} * \vec{r} * A_{34} * M^{-1} * a_4))]\#(w'/X_0) \\
\varepsilon_{X_0c} &= [\vec{r} * ((A_{31} * p + A_{32} * w + A_{33} * c) + (A_{33} * \vec{r} * A_{34} * M^{-1} * a_4))]\#(c'/X_0) \\
\varepsilon_{X_0w_0} &= [-(2/w_0) * ((r * ((1/2) * (p' * A_{11} * p + w' * A_{22} * w + c' * A_{33} * c)) \\
&\quad + p' * A_{12} * w + p' * A_{13} * c + c' * A_{32} * w - \vec{r} * a_4 * ((1/w_0) \\
&\quad * M^{-1} * A_{34} * \vec{r} * (A_{31} * p + A_{32} * w + A_{33} * c))] * (w_0/X_0)
\end{aligned}
$$

Table B.1: Supply elasticities of outputs with respect to output prices[a]

Depend. variable	with respect to							
	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
Y_1	0.0559	-0.0676	-0.0596	-0.0104	0.0088	-0.0234	0.0384	-0.0278
	0.0959	-0.1162	-0.1431	0.0183	0.0008	0.0217	-0.0267	0.0136
Y_2	-0.1235	0.1492	0.1871	-0.0242	-0.0134	0.0007	0.0035	0.0049
	-0.1373	0.1675	0.1638	-0.0012	0.0032	-0.0117	0.0086	0.0005
Y_3	-0.1936	0.2247	0.4247	-0.0311	-0.0442	-0.0631	-0.0365	-0.0077
	-0.3805	0.4592	0.6832	-0.0898	-0.0528	-0.0735	0.0779	0.0295
Y_4	-0.0346	0.0488	-0.1088	0.0945	0.0402	0.0180	-0.0046	-0.0509
	0.0507	-0.0631	-0.0484	0.1080	0.0246	-0.0795	-0.0127	0.0364
Y_5	0.0618	-0.0652	-0.3144	0.1163	0.1171	-0.0332	0.1128	-0.0905
	0.0132	0.0002	-0.3244	0.1154	0.1359	-0.0162	0.1348	0.0003
Y_6	0.0810	-0.0979	-0.1070	-0.0404	-0.0166	0.0633	-0.0267	0.0175
	0.0122	-0.0073	-0.1493	-0.0574	0.0000	0.1349	-0.0122	-0.0131
Y_7	-0.2758	0.3707	-0.0478	-0.1696	0.2896	-0.1639	0.8250	-0.2394
	-0.1188	0.1824	-0.3971	0.0023	0.2793	-0.0946	0.6102	-0.1191
Y_8	0.0195	-0.0454	0.1011	0.2498	-0.0873	-0.0081	-0.4224	1.9115
	-0.0349	0.0221	0.1873	0.2381	-0.0760	-0.0567	-0.3582	1.8832

[a]See Table 4.1 for variable definition. Short run elasticities are provided first, followed immediately by the respective long run elasticities.

Table B.2: Supply elasticities of outputs with respect to variable input prices[a]

Depend. variable	with respect to				
	w_1	w_2	w_3	w_4	w_0
Y_1	-0.0205	0.0644	0.0021	0.0116	0.0158
	-0.0134	-0.0268	-0.0253	0.1004	0.1091
Y_2	0.0578	-0.0092	0.0257	-0.1081	-0.0933
	0.0068	-0.0111	0.0149	-0.1112	-0.1487
Y_3	-0.0816	-0.0856	0.1609	-0.1451	-0.0838
	0.3398	0.1166	0.0282	-0.4053	-0.4998
Y_4	0.1343	-0.0435	-0.1310	-0.2000	-0.0338
	0.0780	-0.1361	-0.0559	-0.1008	0.0659
Y_5	-0.0967	-0.1772	-0.0309	-0.2263	0.4265
	-0.0084	-0.1312	-0.0952	-0.1999	0.1851
Y_6	-0.0622	0.0146	0.0263	0.1866	0.0350
	-0.0275	0.0763	-0.0208	0.1427	-0.0645
Y_7	0.1257	0.3136	0.0894	-1.0081	0.3968
	0.2419	0.2207	-0.1980	-0.7214	-0.3429
Y_8	-0.8498	-0.2941	0.2015	1.5481	-3.4820
	-0.8068	-0.1906	0.1915	1.4314	-3.6696

[a]See Table 4.1 for variable definition. Short run elasticities are provided first, followed immediately by the respective long run elasticities.

Table B.3: Supply elasticities of outputs with respect to user costs of quasi-fixed inputs[a]

Depend. variable	with respect to						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Y_1	-0.0531	0.0212	-0.0081	-0.0027	0.0047	0.0317	0.0183
	0.0318	-0.0330	0.0212	0.0053	0.0002	-0.0173	-0.0162
Y_2	-0.0510	0.0637	-0.0252	-0.0051	-0.0032	-0.0056	-0.0310
	0.0462	-0.0134	-0.0116	-0.0034	-0.0033	0.0024	0.0390
Y_3	-0.0319	0.0403	0.0148	0.0040	-0.0002	0.0036	-0.0689
	-0.3392	0.2474	-0.2352	-0.0101	0.0107	0.0292	0.0634
Y_4	0.1893	-0.1345	0.0467	0.0435	-0.0404	-0.0167	0.1846
	0.0721	-0.0499	0.0086	-0.0043	0.0190	-0.0029	0.0898
Y_5	0.2823	-0.0988	0.0400	0.0503	-0.0335	-0.0091	-0.0307
	0.2187	-0.0320	-0.0007	-0.0256	0.0006	0.0069	0.0222
Y_6	-0.0202	0.0225	-0.0071	-0.0113	0.0097	-0.0051	-0.0620
	0.0300	0.0141	0.0045	-0.0078	-0.0172	-0.0083	-0.0290
Y_7	-0.5762	0.7878	-0.3016	-0.0739	-0.0061	0.1784	-0.5172
	-0.0933	0.0337	-0.1446	-0.0182	-0.0146	0.0390	-0.0329
Y_8	0.4259	-0.1942	-0.1007	-0.1118	0.1129	-0.1673	1.1950
	0.3546	-0.1556	-0.1355	-0.1202	0.1180	-0.1173	1.2974

[a]See Table 4.1 for variable definition. Short run elasticities are provided first, followed immediately by the respective long run elasticities.

Table B.4: Demand elasticities of variable inputs with respect to output prices[a]

Depend. variable	with respect to							
	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
X_1	0.0212	0.0223	0.0246	-0.0714	0.0158	0.0138	0.0374	-0.0424
	-0.0128	0.0140	0.0183	-0.0196	-0.0028	0.0084	-0.0040	0.0348
X_2	-0.0346	0.0429	0.0091	0.0035	0.0116	0.0346	-0.0027	-0.0692
	-0.0191	0.0217	0.0685	0.0465	0.0101	-0.0425	-0.0040	0.0378
X_3	0.1512	-0.1788	-0.2295	0.0875	-0.0005	-0.0376	-0.0058	-0.1111
	0.0619	-0.0582	-0.2929	0.0930	0.0357	0.0257	0.0163	0.0027
X_4	-0.1644	0.1996	0.2723	0.0647	0.0179	-0.1002	0.0734	-0.0485
	-0.1753	0.2145	0.2135	0.0927	0.0398	-0.0821	0.0563	-0.0398
X_0	-0.1913	0.2357	0.1557	-0.0834	-0.0269	0.1010	-0.0235	-0.0298
	-0.4076	0.5070	0.5552	-0.0864	-0.0488	-0.0106	0.1087	0.1919

[a]See Table 4.1 for variable definition. Short run elasticities are provided first, followed immediately by the respective long run elasticities.

Table B.5: Demand elasticities of variable inputs with respect to variable input prices[a]

Depend. variable	with respect to				
	w_1	w_2	w_3	w_4	w_0
X_1	-0.4704	-0.0334	0.2731	0.1000	2.6775
	-0.1900	0.0327	0.0877	0.0904	2.4333
X_2	-0.0056	-0.1446	0.0531	-0.1237	4.9308
	0.0677	-0.1878	0.0193	-0.0970	4.8013
X_3	0.5376	0.0613	-0.3968	-0.0791	-1.9675
	0.6606	0.1341	-0.5136	-0.0519	-2.3503
X_4	0.3948	-0.0699	-0.0702	-0.5634	0.2439
	0.2617	-0.1705	-0.0365	-0.5214	0.3080
X_0	-0.1349	0.1720	0.0266	0.0735	-1.3008
	0.6676	0.4555	-0.2947	-0.2386	-3.0851

[a]See Table 4.1 for variable definition. Short run elasticities are provided first, followed immediately by the respective long run elasticities.

Table B.6: Demand elasticities of variable inputs with respect to user costs of quasi-fixed inputs[a]

Depend. variable	with respect to c_1	c_2	c_3	c_4	c_5	c_6	c_7
X_1	0.0406	0.1805	0.0165	-0.0119	-0.0069	0.0296	-0.3204
	0.0402	-0.0234	-0.0241	0.0005	0.0036	-0.0075	-0.0256
X_2	0.0634	0.0243	0.0394	0.0314	-0.0344	-0.0092	-0.0996
	0.0182	0.0299	-0.0343	-0.0079	0.0162	-0.0037	-0.0212
X_3	0.0394	-0.1694	0.0098	0.0866	-0.0303	-0.0247	0.2300
	0.0152	-0.0993	-0.0151	-0.0148	0.0010	-0.0085	0.3301
X_4	-0.1536	0.0364	-0.0495	0.0053	-0.0004	0.0134	0.1476
	0.0026	0.0298	-0.0265	0.0040	-0.0043	0.0019	0.0808
X_0	0.1176	0.0329	0.0508	0.0387	-0.0578	0.0000	0.0357
	-0.3778	0.2138	-0.3680	-0.0370	0.0294	0.0347	0.4854

[a]See Table 4.1 for variable definition. Short run elasticities are provided first, followed immediately by the respective long run elasticities.

Table B.7: Demand elasticities of quasi-fixed input stocks with respect to output prices[a]

Depend. variable	with respect to							
	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
Z_1	0.0246	-0.0318	0.0317	-0.0253	-0.0258	0.0069	-0.0075	0.0069
	0.0404	-0.0649	0.5408	-0.3199	-0.1629	-0.0827	0.1762	-0.1028
Z_2	0.0038	-0.0035	-0.0297	0.0288	0.0054	-0.0124	-0.0091	0.0063
	0.2076	-0.2486	-0.5228	0.2501	0.0487	-0.0202	-0.2068	0.0438
Z_3	-0.0420	0.0524	0.0622	-0.0203	-0.0022	-0.0007	0.0287	0.0045
	-3.8697	4.8474	5.4483	-1.4657	-0.1726	-0.1235	2.4597	0.6166
Z_4	-0.0716	0.0952	-0.0301	-0.0370	0.0271	0.0619	0.0348	0.0556
	-0.2848	0.3781	-0.1355	-0.1535	0.1047	0.2668	0.1319	0.1618
Z_5	-0.1865	0.2442	-0.1404	-0.0839	0.0414	0.2447	0.0357	-0.2070
	-1.7319	2.2700	-1.1644	-0.6937	0.3985	2.0930	0.3675	-1.4858
Z_6	0.0497	-0.0609	-0.0790	0.0256	-0.0147	0.0415	-0.0670	0.0414
	0.3772	-0.4544	-0.8986	0.3652	-0.0352	0.3645	-0.6185	0.3776
Z_7	0.0117	-0.0164	0.0034	-0.0445	0.0010	0.0176	0.0109	-0.0315
	0.1345	-0.1747	-0.0650	-0.1273	-0.0087	0.0319	-0.0037	-0.1073

[a]See Table 4.1 for variable definition. Short run elasticities are provided first, followed immediately by the respective long run elasticities.

Table B.8: Demand elasticities of quasi-fixed input stocks with respect to variable input prices[a]

Depend. variable	with respect to				
	w_1	w_2	w_3	w_4	w_0
Z_1	0.1160	0.0425	-0.0204	-0.0035	0.0169
	0.7195	0.5417	0.0526	-0.3110	-0.0076
Z_2	0.0317	0.0348	-0.0600	0.0018	-0.1011
	-0.2543	-0.0352	-0.2941	0.4147	-0.0870
Z_3	0.0777	0.0392	-0.0144	-0.0537	-0.0634
	7.9451	3.8426	-2.1080	-5.1841	-7.3765
Z_4	-0.0066	0.0273	-0.0046	0.0428	-0.1207
	-0.0607	0.1110	-0.0041	0.1436	-0.3930
Z_5	-0.0600	0.1311	-0.0322	-0.1413	0.0226
	-0.1827	1.2356	-0.4818	-1.1844	-0.5627
Z_6	0.0457	-0.0640	-0.0454	0.0883	0.0770
	0.0095	-0.7623	-0.3964	0.8554	0.5974
Z_7	-0.1726	-0.0761	0.1291	0.0549	0.2602
	-0.6133	-0.2312	0.4124	0.2796	0.9194

[a]See Table 4.1 for variable definition. Short run elasticities are provided first, followed immediately by the respective long run elasticities.

Table B.9: Demand elasticities of quasi-fixed input stocks with respect to user costs of quasi-fixed inputs [a]

Depend. variable	with respect to						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Z_1	-0.1380	0.0397	-0.0161	0.0016	0.0013	-0.0092	-0.0105
	-1.3585	0.6811	-0.2292	-0.0273	0.0235	0.1008	-0.2120
Z_2	0.0727	-0.1455	0.0161	0.0078	0.0007	-0.0015	0.1531
	0.9477	-1.3755	0.2999	0.0796	-0.0038	-0.0995	0.8586
Z_3	-0.0773	0.0655	-0.0591	-0.0042	0.0024	0.0063	-0.0019
	-6.5523	5.0648	-5.2388	-0.3272	0.2104	0.5412	1.4242
Z_4	-0.0506	0.1429	-0.0312	-0.0775	0.0193	0.0149	-0.0926
	-0.1467	0.5248	-0.0953	-0.2684	0.0484	0.0495	-0.3807
Z_5	0.2232	-0.0192	0.0819	0.0772	-0.1264	-0.0252	-0.0774
	1.6946	0.0411	0.5416	0.4533	-0.9726	-0.1687	-0.4503
Z_6	0.0568	-0.0820	0.0289	0.0106	-0.0037	-0.0499	0.0015
	1.1336	-1.0227	0.3471	0.0990	-0.0402	-0.4443	0.1508
Z_7	-0.0194	0.1337	0.0025	-0.0058	-0.0031	0.0008	-0.2568
	-0.0243	0.2467	0.0914	-0.0142	-0.0022	-0.0014	-0.7432

[a]See Table 4.1 for variable definition. Short run elasticities are provided first, followed immediately by the respective long run elasticities.